**Chapter 5: Electromagnetic Optics**

* P5-1. **An Electromagnetic Wave.** An electromagnetic wave in free space has an electric-field vector \( \mathbf{E} = f(t-z/c_0)\mathbf{\hat{x}} \) where \( \mathbf{\hat{x}} \) is a unit vector in the \( x \) direction, and
\[
f(t) = \exp(-t^2/\tau^2)\exp(j2\pi v_c t),
\]
where \( \tau \) is a constant. Describe the physical nature of this wave and determine an expression for the magnetic-field vector.

* P5-2. **Dielectric Media.** Identify the media described by the following equations, regarding linearity, dispersion, isotropy, and homogeneity. Assume that all media are spatially nondispersive.

  a) \( P = \varepsilon_0 \chi E - a \nabla \times E \)

  b) \( P + aP^2 = \varepsilon_0 E \)

  c) \( a_1 \frac{\partial^2 P}{\partial t^2} + a_2 \frac{\partial P}{\partial t} + P = \varepsilon_0 \chi E \)

  d) \( P = \varepsilon_0 \left(a_1 + a_2 \exp\left[-(x^2 + y^2)\right]\right)E \)

P5-3. A 3V-torch is driven by a current of 0.25 A. About one per cent of the power is converted to light. Assume that the light is monochromatic with the wavelength 550 nm. How many photons are emitted each second?

* P5-4. An isotropic and approximately monochromatic point source transmits light in air with a power of 100 W.

  a) Calculate the intensity at the distance 1.0 m.

  b) Calculate the amplitude of the E- and the B-field.

* P5-5. In a microwave oven the frequency is always 2.45 GHz (this is a frequency that is absorbed by the water molecules in the food). The transmitter in an ordinary microwave oven has a power of 750 W. Estimate the maximum electric field inside the microwave oven assuming that the effect is distributed on an area of \( 0.10 \text{ m}^2 \).

P5-6. A cell phone that transmits at the frequency 900 MHz releases a power of 2.0 W. Assume that the antenna of the phone distribute the microwaves evenly in all directions. Calculate the amplitude of the electric field and the magnetic density at the ear 5.0 cm from the antenna.

* P5-7. A plane, harmonic wave is propagation through glass. The electric field is given by:
\[
E_z = E_0 \cdot \cos\left(\pi \cdot 10^{15} \left(t - \frac{x}{0.65 \cdot c}\right)\right)
\]
Calculate

  a) the frequency

  b) the wavelength in vacuum

  c) the wavelength in glass

  d) the propagation velocity in glass
e) the refractive index

* P5-8. Laser light propagates in a transparent nonmagnetic material (\( \mu_r = 1.000 \)). The electric field of the laser light varies according to

\[
E = E_0 \sin(\omega t - kx)
\]

where \( E_0 = 3.5 \text{ kV/m} \), \( \omega = 2.272 \cdot 10^{15} \text{ s}^{-1} \) and \( k = 1.287 \cdot 10^7 \text{ m}^{-1} \).

a) What is the wavelength of the laser light inside the material?
b) What is the refractive index in the material where the wave propagates?
c) Calculate the amplitude of the magnetic density in the material.
d) Calculate the intensity of the laser light.

* P5-9.

a) Make an estimation of the dispersion coefficient \( D_\lambda \) of quarts at the wavelength 852 nm by using the following table:

<table>
<thead>
<tr>
<th>Wavelength/nm</th>
<th>Refractive index</th>
</tr>
</thead>
<tbody>
<tr>
<td>707</td>
<td>1.45515</td>
</tr>
<tr>
<td>852</td>
<td>1.45247</td>
</tr>
<tr>
<td>997</td>
<td>1.45043</td>
</tr>
</tbody>
</table>

b) What would the broadening of a pulse at 852 nm with the spectral width 80 nm be at a distance of 12 km?

Hint: Make a numerical calculation of the second derivative of the refractive index in terms of the wavelength.
Answers chapter 5:

P5-1: This is a pulse of width $2 \tau$. The pulse is moving in the positive $z$-direction and the light is linearly polarized along the $x$-axis. $H_0 = f(t - \frac{z}{c_0}) \sqrt{\frac{\epsilon}{\mu}} \hat{y}$

P5-2:

<table>
<thead>
<tr>
<th>Relation</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) $P = \epsilon_0 \chi E - a \nabla \times E$</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>b) $P + aP^2 = \epsilon_0 E$</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>c) $a_1 \dot{\nabla}^2 P + a_2 \frac{\partial P}{\partial t} + P = \epsilon_0 \chi E$</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>d) $P = \epsilon_0 \left( a_1 + a_2 \exp\left[-\left(x^2 + y^2\right)\right]\right) E$</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>

1) Linearity 2) Dispersion 3) Isotropy 4) Homogeneity

P5-3: $2 \cdot 10^{16}$ photons/s

P5-4: a) 7.96 W/m² b) 77.4 V/m and $2.58 \cdot 10^{-7}$ Vs/m²

P5-5: 2.4 kV/m

P5-6: 0.22 kV/m and 0.73 μT

P5-7: a) $5 \cdot 10^{14}$ Hz b) 0.6 μm c) 0.39 μm d) $1.95 \cdot 10^8$ m/s e) 1.538

P5-8: a) 488.2 nm b) $n = 1.698$ c) 20 μT d) 28 kW/m²

P5-9: a) $8.46 \cdot 10^{-5}$ s b) 81 ns