



Photo:
Tomas Svensson

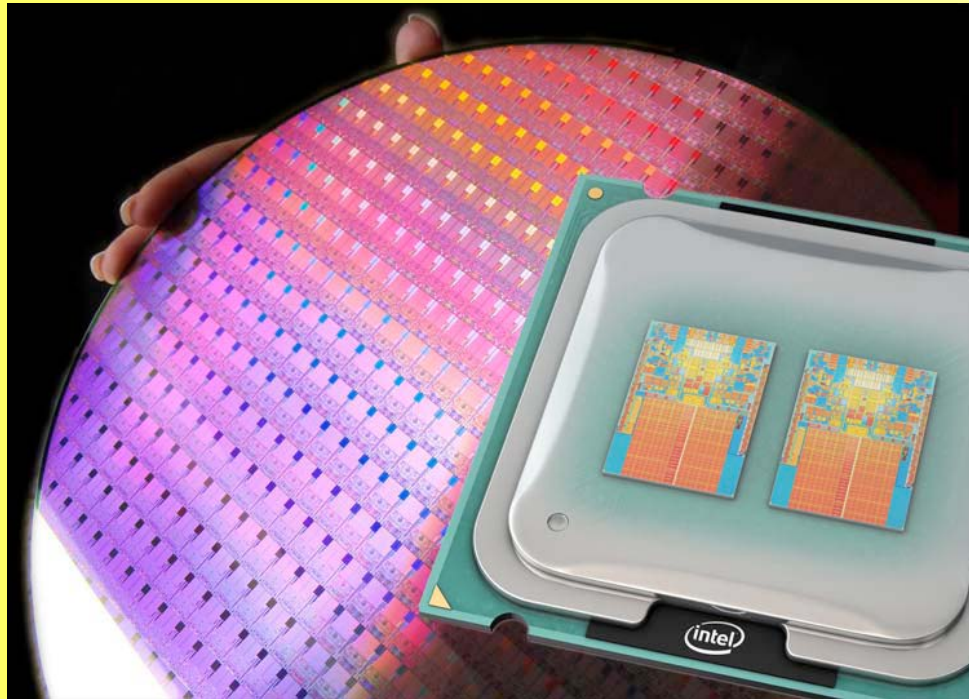
Quantum computing and the Quantum control lab

Stefan Kröll



With the shrinking dimensions of the technology of today it will be increasingly important to utilize and master the quantum world when developing the technology of tomorrow

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300 billions of transistors/wafer



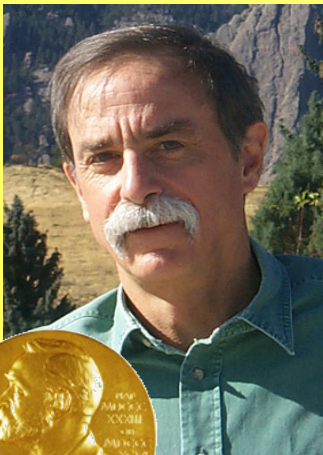
7 billion people

Courtesy: Heiner Linke

Quantum control lab

- Nobel prize 2012: David Wineland, NIST, Boulder, USA Serge Haroche, École Normale Supérieure, Paris

“For ground-breaking experimental methods that enable measuring and manipulation of individual quantum systems”



David Wineland

Outline

- Quantum computing and quantum resources
- Quantum bits (qubits) & the Bloch sphere
- The quantum control lab and the Lund approach to quantum computing



Why is Quantum Computing (QC) interesting?

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- QC is a structured way to learn how to design fully controllable quantum systems
- A quantum computer has the potential to solve certain *computationally hard problems* which are untractable on conventional computers



Computationally hard problems

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- The number of steps required to solve the problem using the best known algorithms on classical computers increase exponentially with the size of the problem
- For some of these problems there are, however, quantum algorithms where the number of steps only increase polynomially



Computationally hard problems

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- Consider a computationally hard problem with an input represented by $n=25$ bits that takes 1 hour to solve on a classical computer (computation time goes as 2^n)
- How long time would it take to solve a problem requiring $n=50$ bits?
 - 1000 years!
- While on a quantum computer (if computation time goes as n^2) time would increase from 1 hour to 4 hours
- It is good to increase computer speed but it is even better to decrease algorithm complexity



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Quantum resources



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In quantum computers data is represented by quantum bits (qubits)

- A qubit is a quantum mechanical systems with two states $|0\rangle$ and $|1\rangle$ that can be in any arbitrary superposition

$$\Psi = \alpha|0\rangle + \beta |1\rangle \quad \text{Eq. 13.1}$$



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Superposition of states makes a quantum computer powerful

Input data (4 qubits)

$$(|0\rangle + |1\rangle) / \sqrt{2} 0$$

$$(|0\rangle + |1\rangle) / \sqrt{2} 1$$

$$(|0\rangle + |1\rangle) / \sqrt{2} 1$$

$$(|0\rangle + |1\rangle) / \sqrt{2} 0$$

**Quantum
Computer**

$$\begin{aligned} \text{Input} &\equiv (|0\rangle + |1\rangle) (|0\rangle + |1\rangle) (|0\rangle + |1\rangle) (|0\rangle + |1\rangle) / 4 = \\ &= (|0000\rangle + |0001\rangle + |0010\rangle + |0011\rangle + \dots + |1111\rangle) / 4 \end{aligned}$$



Fourier transforms

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- Fast Fourier transform on a function represented by $N=2^n$ numbers
- Classically this requires $N \log_2(N) = n2^n$ steps
- On QC $[\log_2(N)]^2 = n^2$ steps
- This looks fantastic!
- However, we do not have full information, readout will collapse the state



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Multiple qubits

- n qubits span a computational basis $|x_1, x_2, x_3, \dots, x_n\rangle$
- The quantum state is specified by 2^n amplitudes
- Lets say $n \approx 500$, would it be easy to store these amplitudes in a classical memory?
- The number of amplitudes is larger than the estimated number of atoms in the universe



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Entanglement (13.1.1)

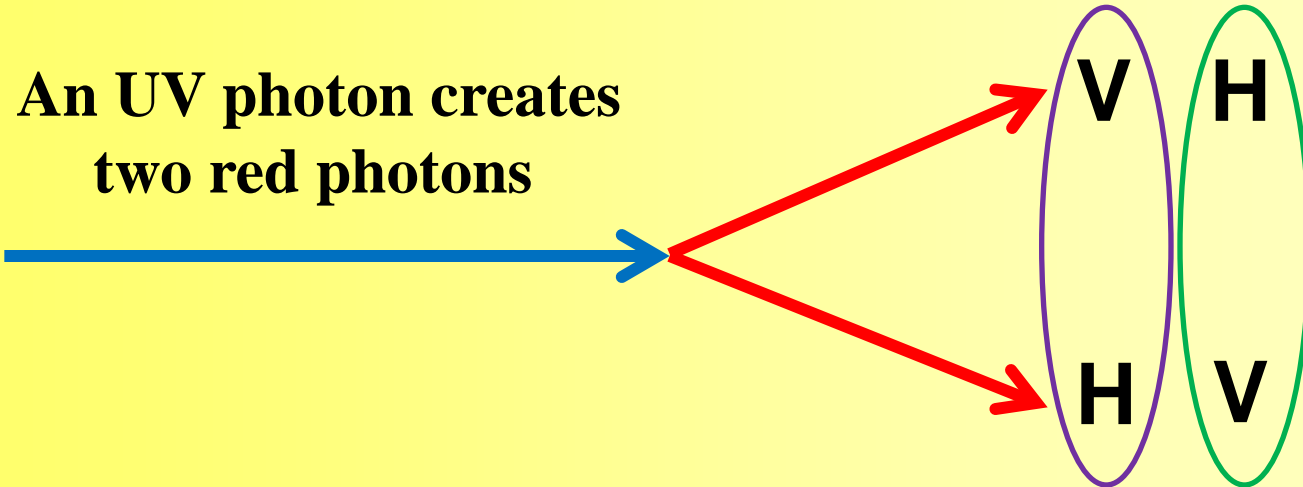
- If two systems, a and b, described by a wave function $\Psi(a,b)$ are entangled then it is not possible to rewrite $\Psi(a,b)$ as a product of two wavefunctions $\Psi_1(a) * \Psi_2(b)$
- EPR paradox & Einsteins objection



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The wave function does not collapse until there is a measurement

An UV photon creates
two red photons



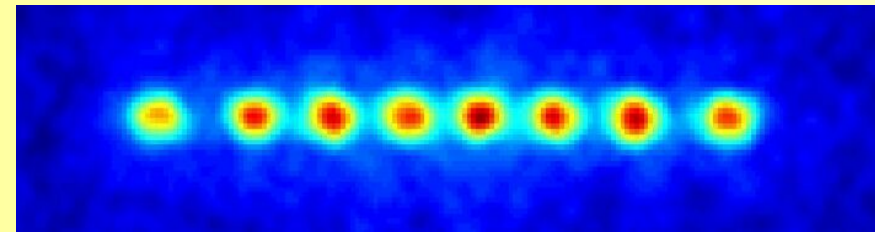
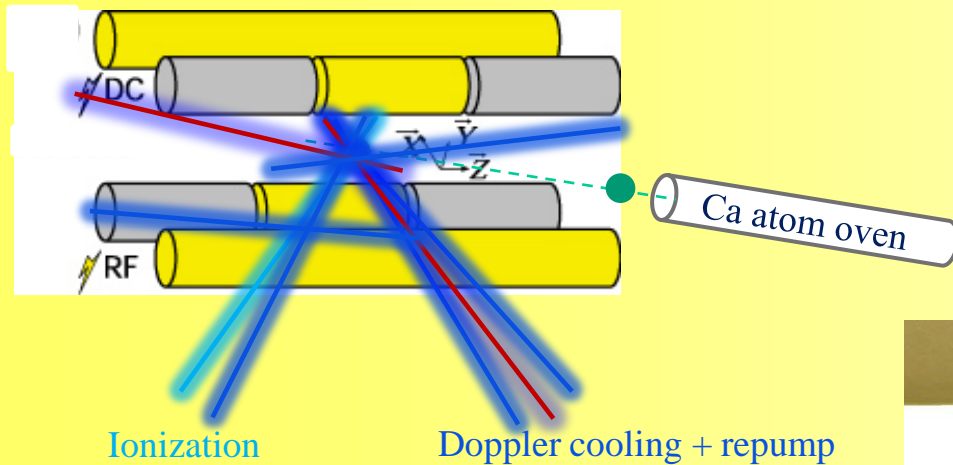
Which one of the red photons that is vertically polarized is not determined until the polarization is measured

Ion trapping

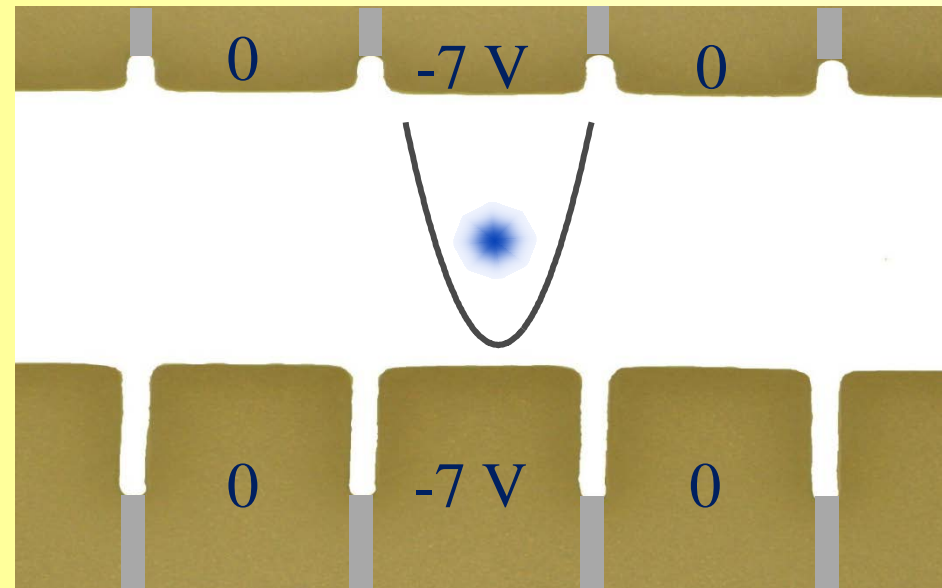
Christopher Monroe



The trapping procedure:

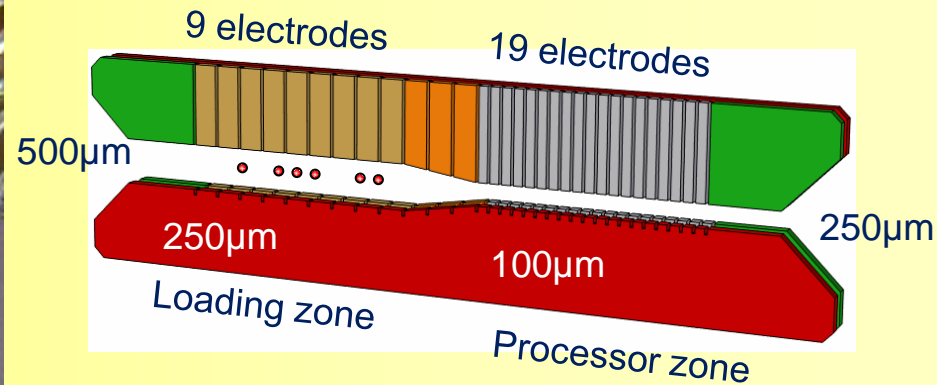
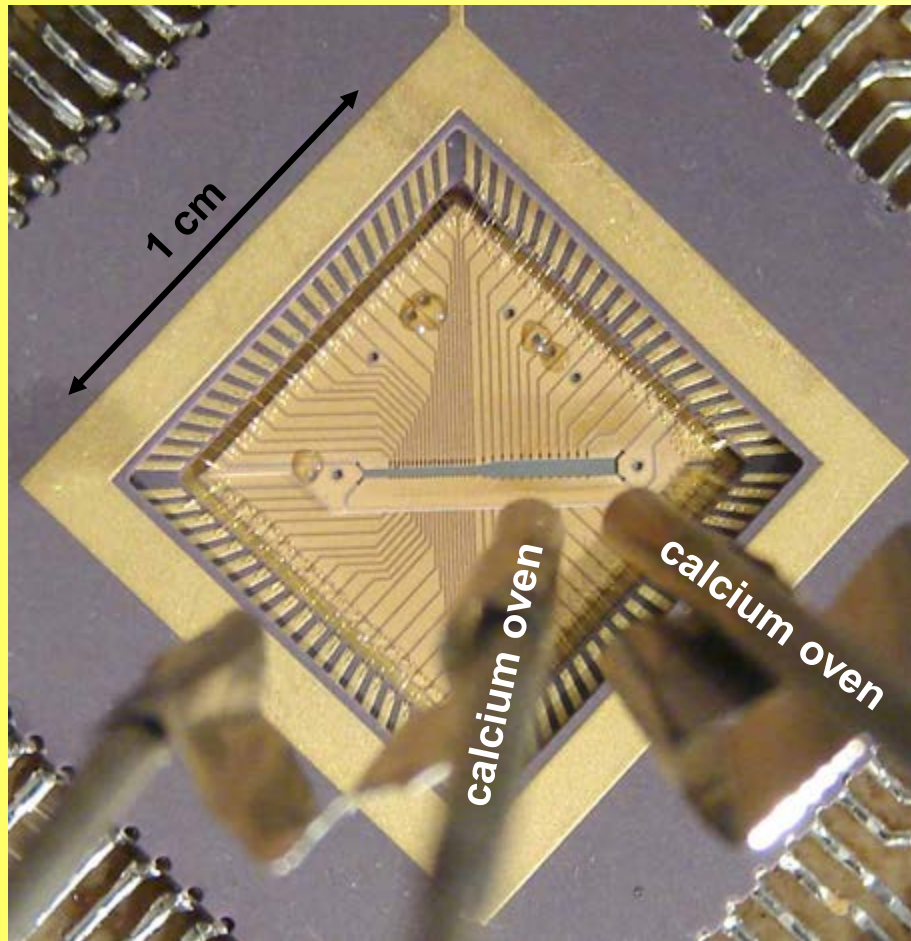


- A full experiment needs:
8 different lasers
(12 different beams)



Courtesy: Andreas Walther

Experimental setup: Segmented micro-trap



Microtraps:

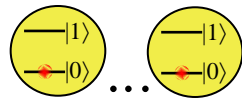
- Segments enable transport/scalability
- Small size necessary for scalability

Schulz, et al., *NJP* **10**,
045007 (2008)

Poschinger et al, *J. Phys.*
B **42**, 154013 (2009)

Courtesy: Andreas Walther

Ion traps:



7) Scalability

D. Kielpinski, C. Monroe, and D. Wineland, *Nature* 417, 709 (2002).

Isaac Chuang, video showing 3 qubit error correction recursive:

The screenshot displays a quantum simulation interface. On the left, a grid of qubits is shown, with some qubits highlighted in green, red, blue, and yellow. A central text box lists requirements for the simulation. On the right, a legend identifies the colors: green for Data, red for Ancilla, blue for Sympathetic, and yellow for Damaged. Below the legend, a terminal window shows simulation statistics.

Requirements:

- Fast, $T_{transp} \sim T_{gate}$
- Superposition states have to survive
- Entangled states have to survive
- Low excess motional excitation

Legend

- Data
- Ancilla
- Sympathetic
- Damaged

```
Time-      0.012111 s
Errors-
      X: 0 Z: 0
      Total: 0
Message:
Connecting for ZZI
Action:
move
```



Why is it difficult to construct a quantum computer

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- The quantum bits must remain in superposition states all through the calculations – Thus the quantum bits must not interact with the environment
- Quantum logics requires that bits can control each other – Thus the quantum bits must interact with each other



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**Why could it still be possible to
make quantum computers?**



Quantum error correction

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- There are efficient error correction algorithms for correcting errors in quantum computer operation when the error per operation is $< 10^{-4}$ ($< 10^{-2}$)
- If quantum operations can be performed with a fractional error of less than 10^{-4} we can keep an arbitrary large quantum system coherent for an arbitrary long time!



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**Will there be quantum
computers?**



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It is difficult to predict technological development, I

- **“This 'telephone' has too many shortcomings to be seriously considered as a means of communication.”**
 - *Western Union internal memo, 1876*

- **“I think there is a world market for maybe five computers.”**
 - *Thomas Watson, chairman of IBM, 1943*



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It is difficult to predict technological development, II

- **”We never experiment with just one electron or atom or (small) molecule. In thought-experiments we sometimes assume that we do; this invariably entails ridiculous consequences ... we are not experimenting with single particles, any more than we can raise *Ichtyosauria* in the zoo”**

– *Erwin Schrödinger*

– *Brit. J. Phil. Sci. 3 (1952) 233.*

Information is physical

Rolf Landauer



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Questions?

Outline

- Quantum computing and quantum resources
- Quantum bits (qubits) & the Bloch sphere
- The quantum control lab and the Lund approach to quantum computing

In quantum computers data is represented by quantum bits (qubits)

- A qubit is a quantum mechanical systems with two states $|0\rangle$ and $|1\rangle$ that can be in any arbitrary superposition

$$\Psi = \alpha|0\rangle + \beta|1\rangle$$

What can be known about α and β ?

Qubit representation (Eq. 13.2)

$$|\Psi\rangle = e^{i\gamma} \left(\cos \frac{\theta}{2} |0\rangle + e^{i\varphi} \sin \frac{\theta}{2} |1\rangle \right)$$

The Bloch sphere

$$|\Psi\rangle = e^{i\gamma} \left(\cos \frac{\theta}{2} |0\rangle + e^{i\varphi} \sin \frac{\theta}{2} |1\rangle \right)$$

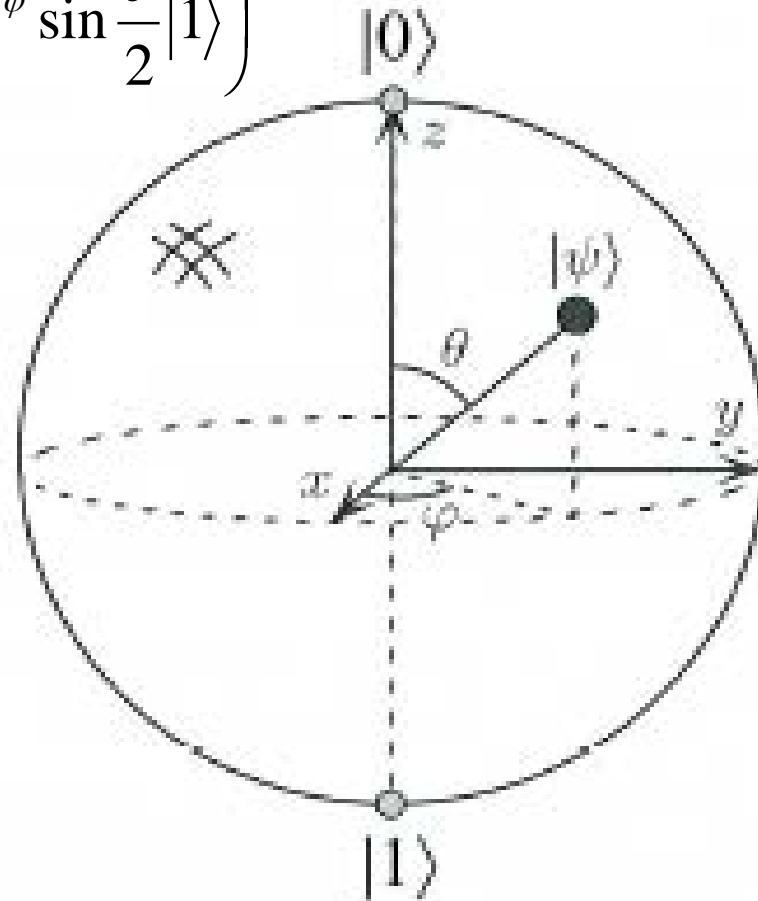
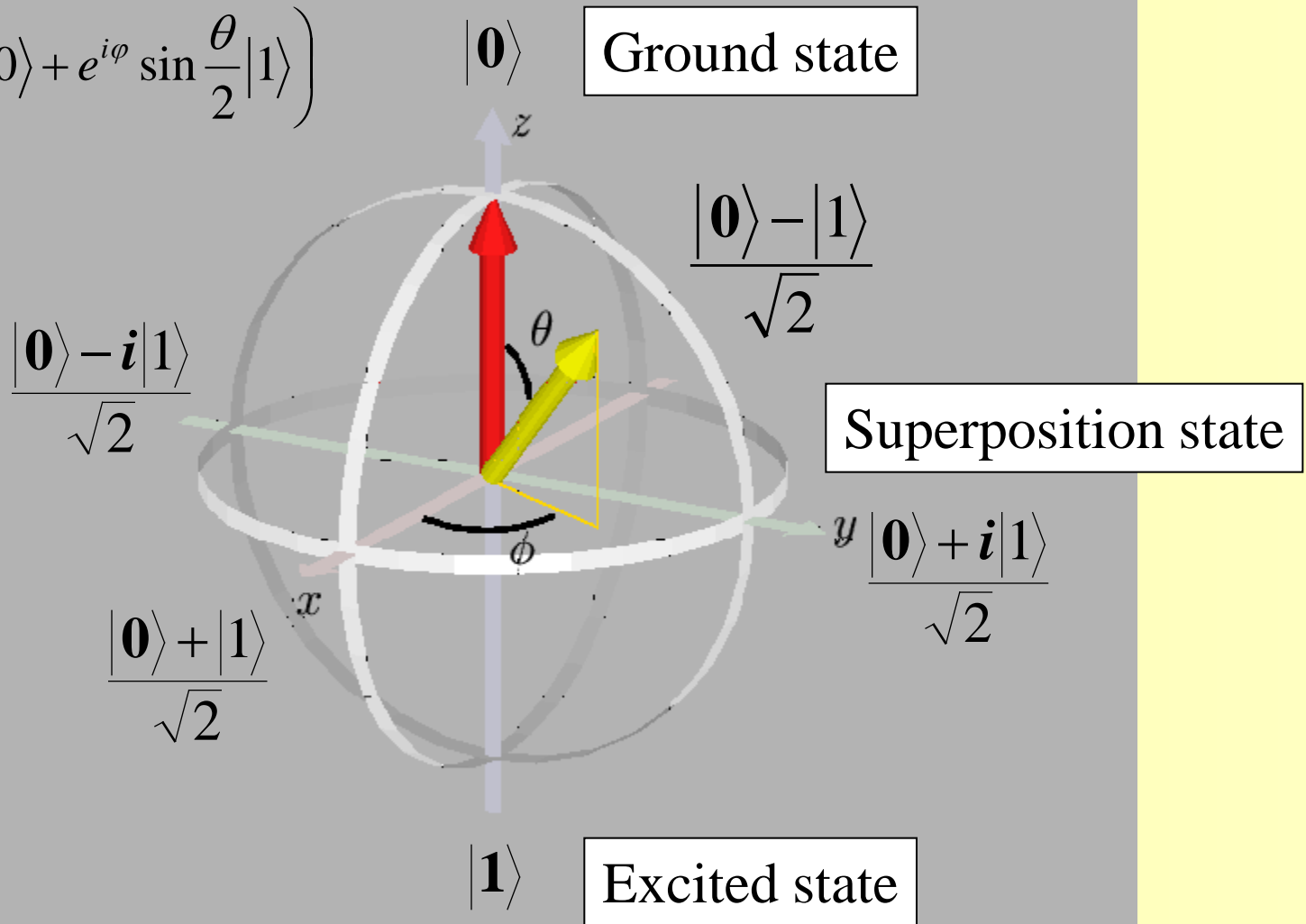


Figure 1.3. Bloch sphere representation of a qubit.

The Bloch sphere

$$|\Psi\rangle = e^{i\gamma} \left(\cos \frac{\theta}{2} |0\rangle + e^{i\phi} \sin \frac{\theta}{2} |1\rangle \right)$$



Outline

- Quantum computing and quantum resources
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What is needed to construct a
quantum computer?



Requirements for quantum computing

The Di Vincenzo criteria
(IBM 2000)

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- Coherent two-state systems acting as qubits
- Possibility to manipulate the qubits individually (single qubit operations)
- Coupling between any two qubits (two-bit gates)
- Possibility for reliable read-out of the individual qubits
- Scalability



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Requirements for quantum computing

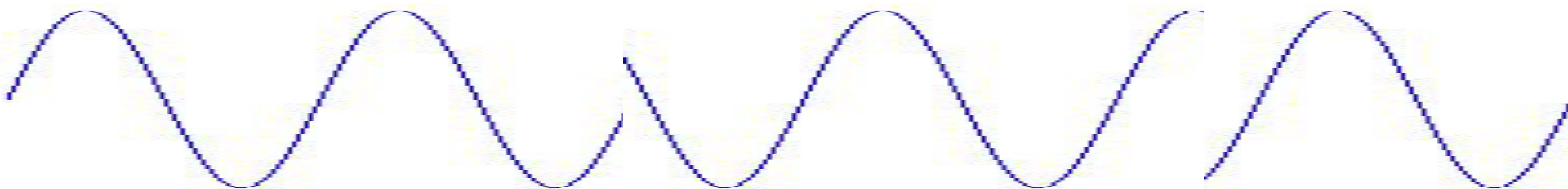
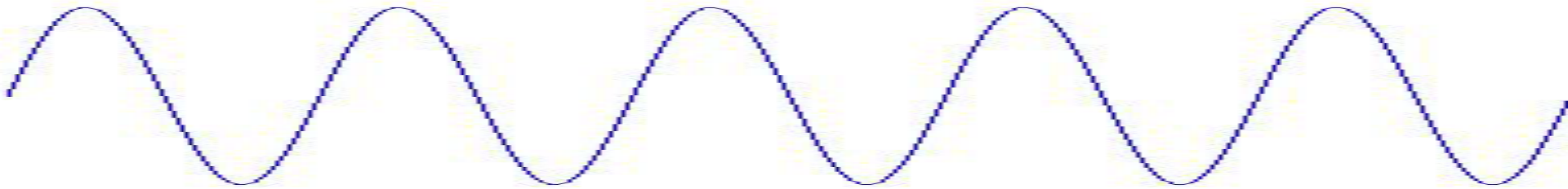
(Di Vincenzo criteria)

- Coherent two-state systems acting as qubits
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Coherence





Quantum coherence

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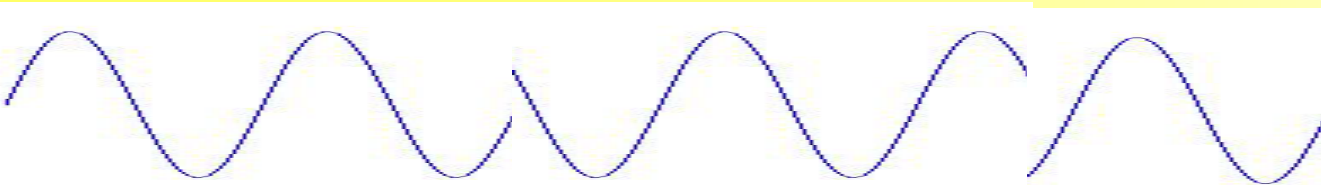
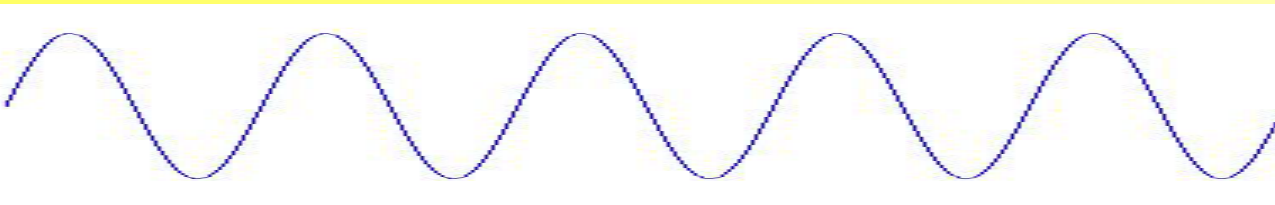
$$\Psi_1(r, t) = e^{-iE_1 t/\hbar} \Psi_1(r) = e^{-iE_0 t/\hbar} \Psi_1(r) e^{-i(E_1 - E_0)t/\hbar}$$

$|1\rangle$ ————— E_1

$$\Psi_{TOT} = e^{-iE_0 t/\hbar} [\Psi_0(r) + \Psi_1(r) e^{-i(E_1 - E_0)t/\hbar}] / \sqrt{2}$$

$|0\rangle$ ————— E_0

$$\Psi_0(r, t) = e^{-iE_0 t/\hbar} \Psi_0(r)$$





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How coherent does the quantum system need to be?

Choosing a good system

$$N_{\text{operation}} = \frac{T_{\text{coherence}}}{T_{\text{single operation}}}$$

Strong interaction

Rare earth doped crystals





Rare-earth-ion doped crystals

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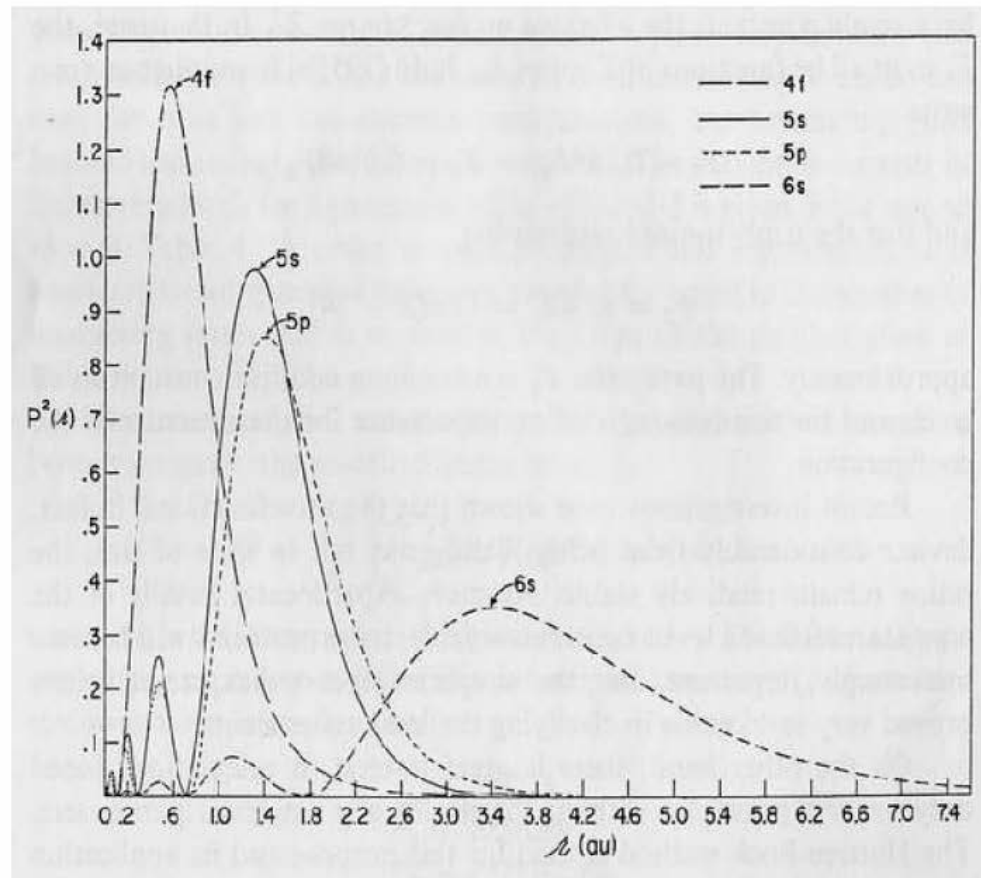
- Rare earth ion doped crystals and glasses
 - Solid state lasers, fibre amplifiers, scintillators, X-ray medical imaging, high energy radiation detectors.

Rare-earth-ion doped inorganic crystals

(Free-atom-like material)

- Outer electron configuration rare earth ions is $4f^n5s^25p^66s^2$
- 4f-4f transitions, i.e., transitions between different electronic states within the 4f-shell are free-atom-like, because 5s, 5p and 6s electrons shield the 4f electrons from the surrounding

5s, 5p and 6s electrons shield the 4f electrons from the surrounding



G. H. Dieke, Spectra and energy levels of Rare earth Ions in Crystals

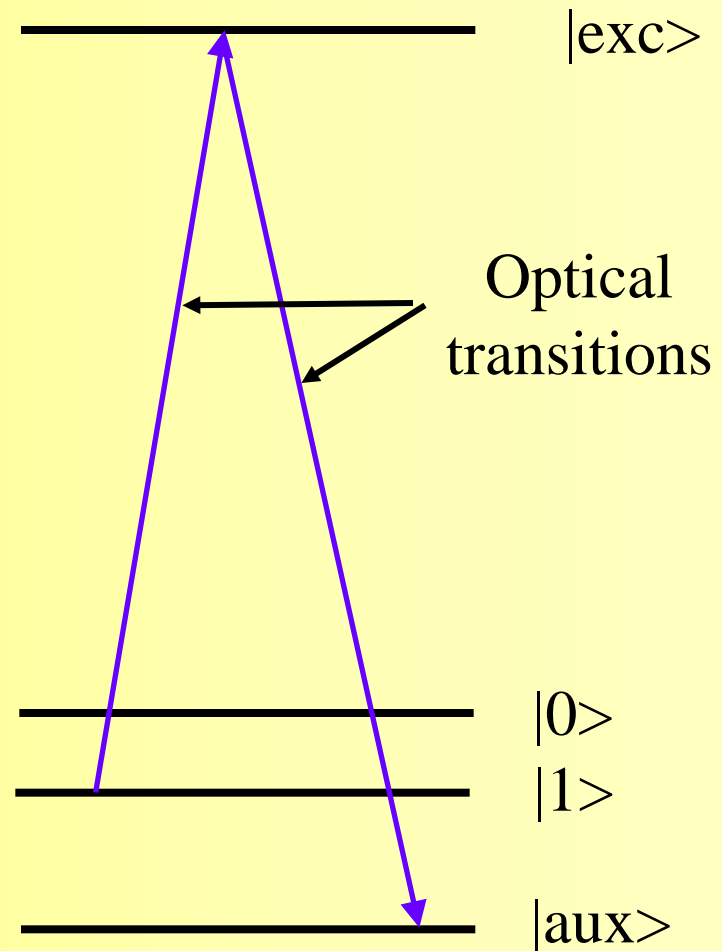


The rare-earth-ions hyperfine states are used as qubit states

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- Long coherence times of the optical transitions (up to several ms)
- At 4 Kelvin the ground state hyperfine levels can have hour long coherence times and lifetimes

- π -pulse takes $< 1 \mu\text{s}$
- Ground state with hyperfine splitting





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Requirements for quantum computing

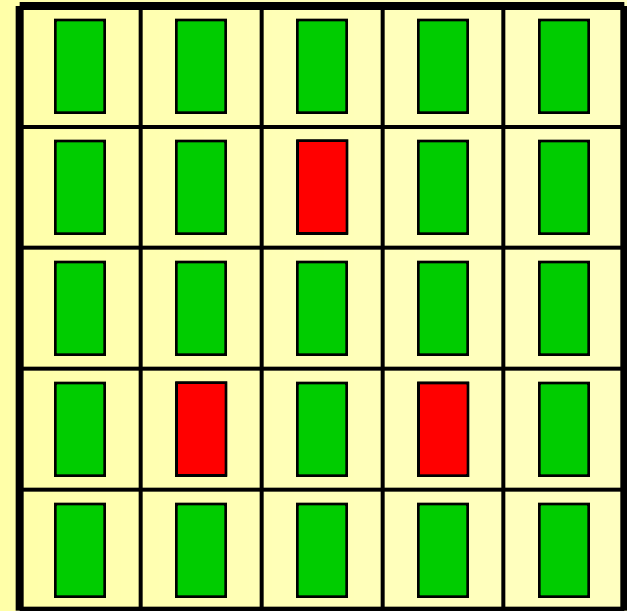
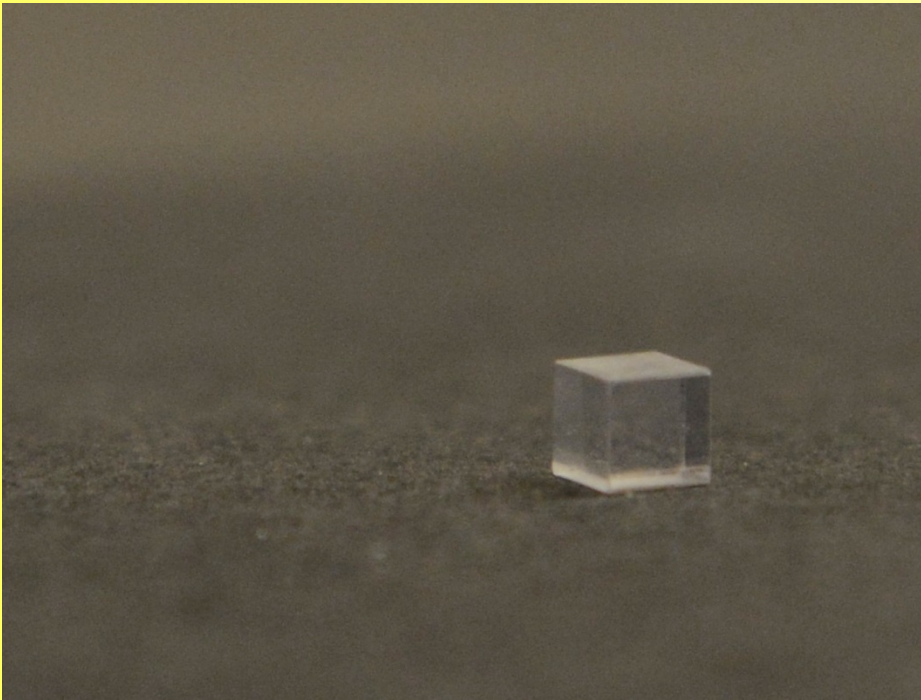
- Coherent two-level systems acting as qubits
- **Possibility to manipulate the qubits individually (one-bit gates)**
- Coupling between any two qubits (two-bit gates)
- Possibility for reliable read-out of the individual qubits
- Scalability



Crystal structure

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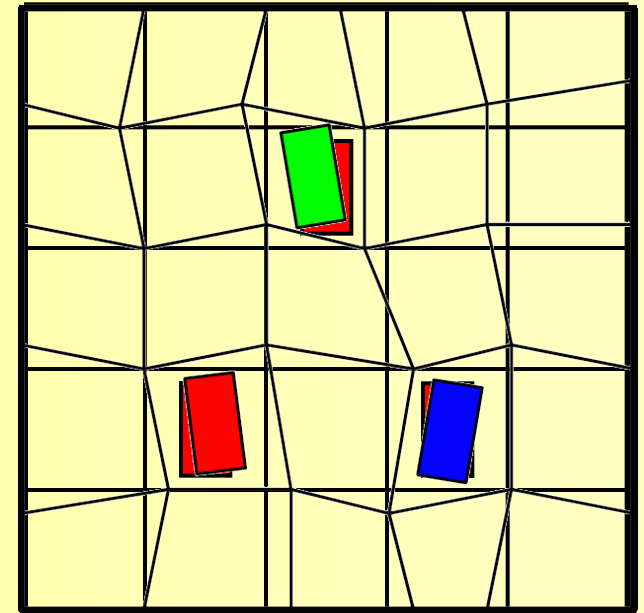
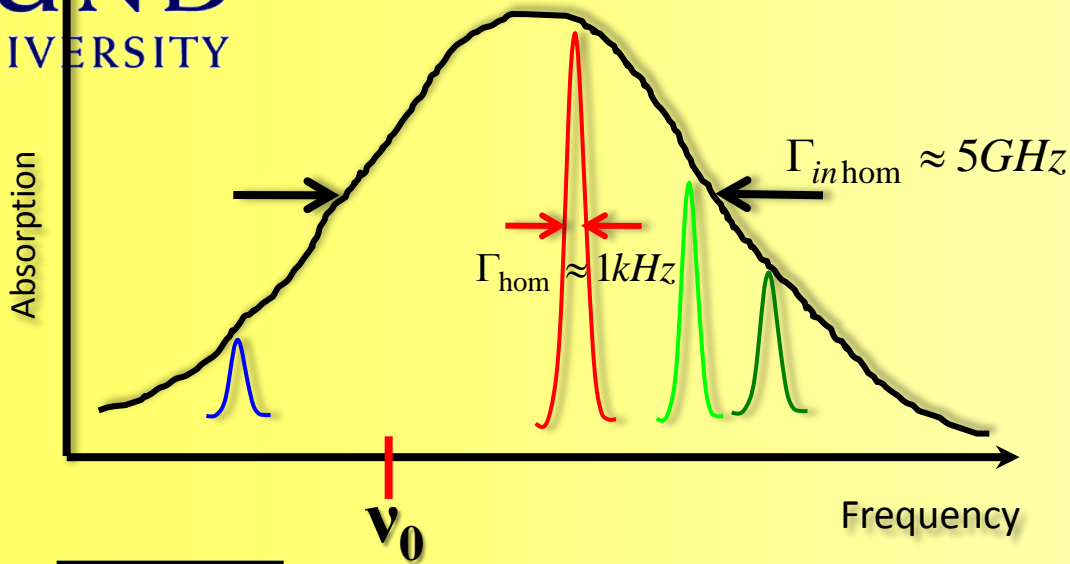
Conceptual picture
of crystal





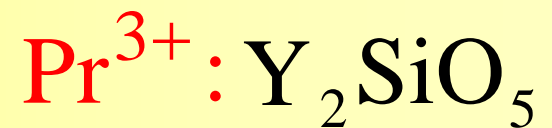
Engineering the absorption structure

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Optical pumping

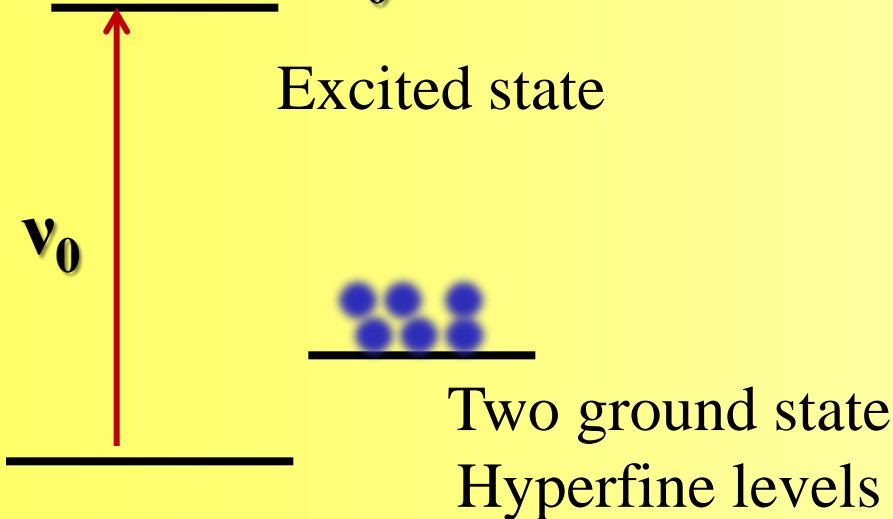
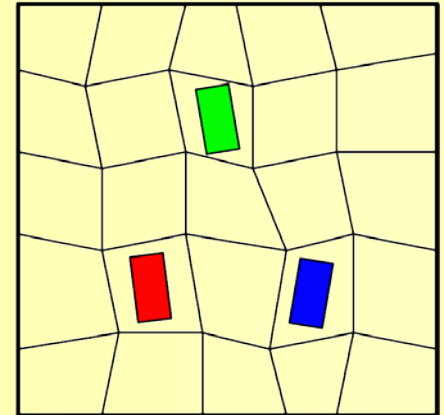
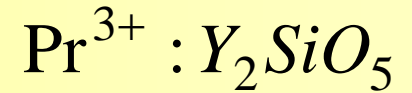
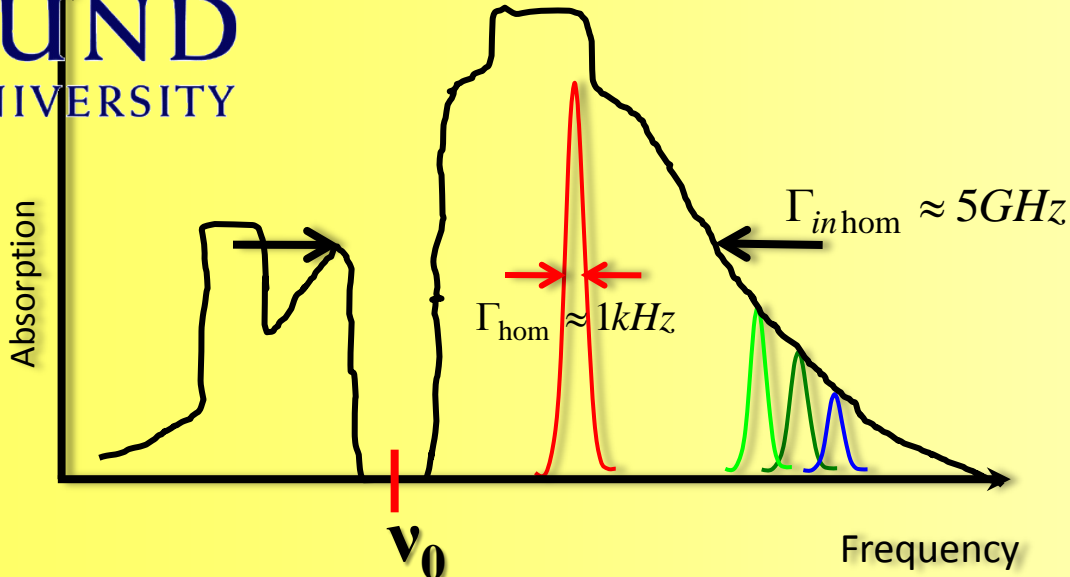
Two ground state
Hyperfine levels





Engineering the absorption structure

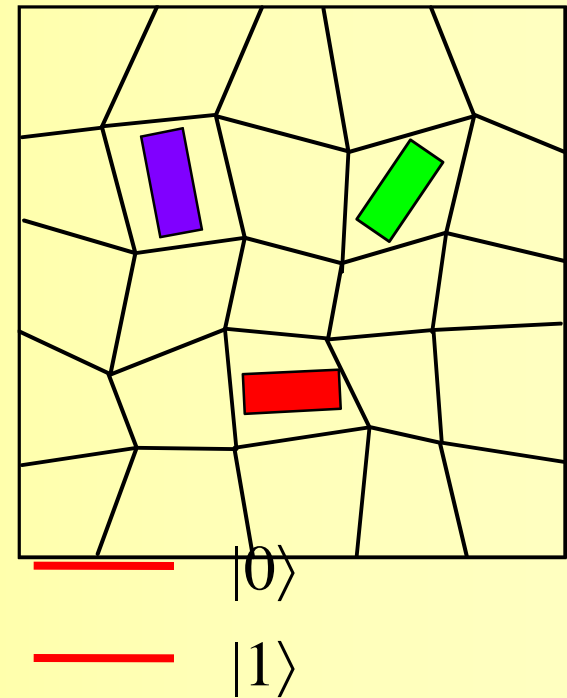
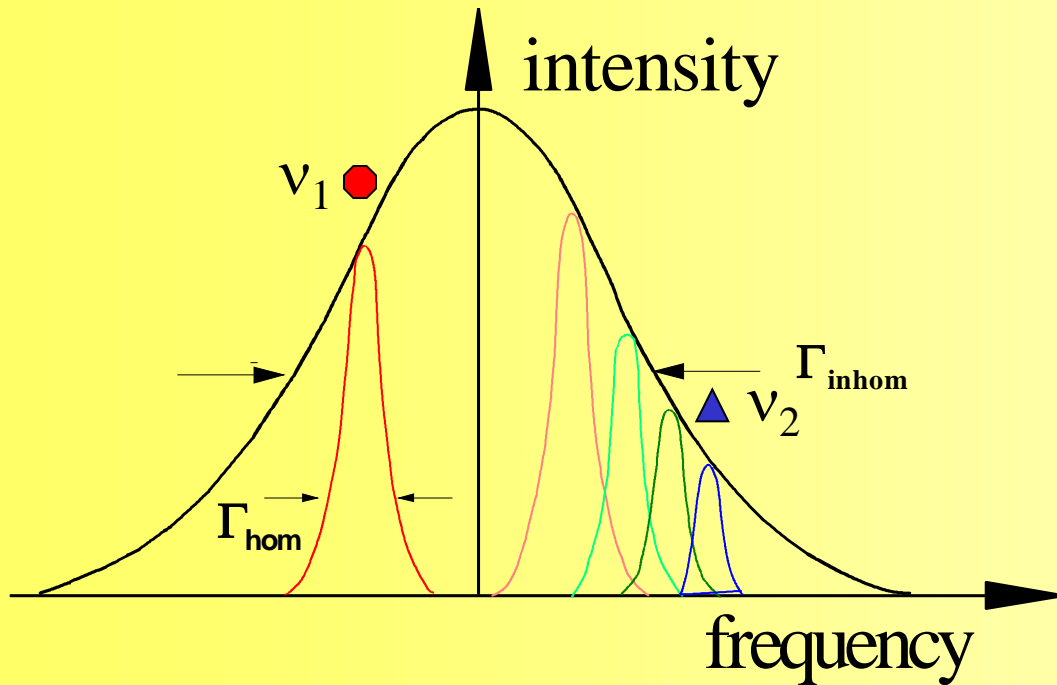
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Absorption line from dopant ions in a rare earth doped inorganic crystal

Conceptual picture of crystal with dopant ions $|exc\rangle$

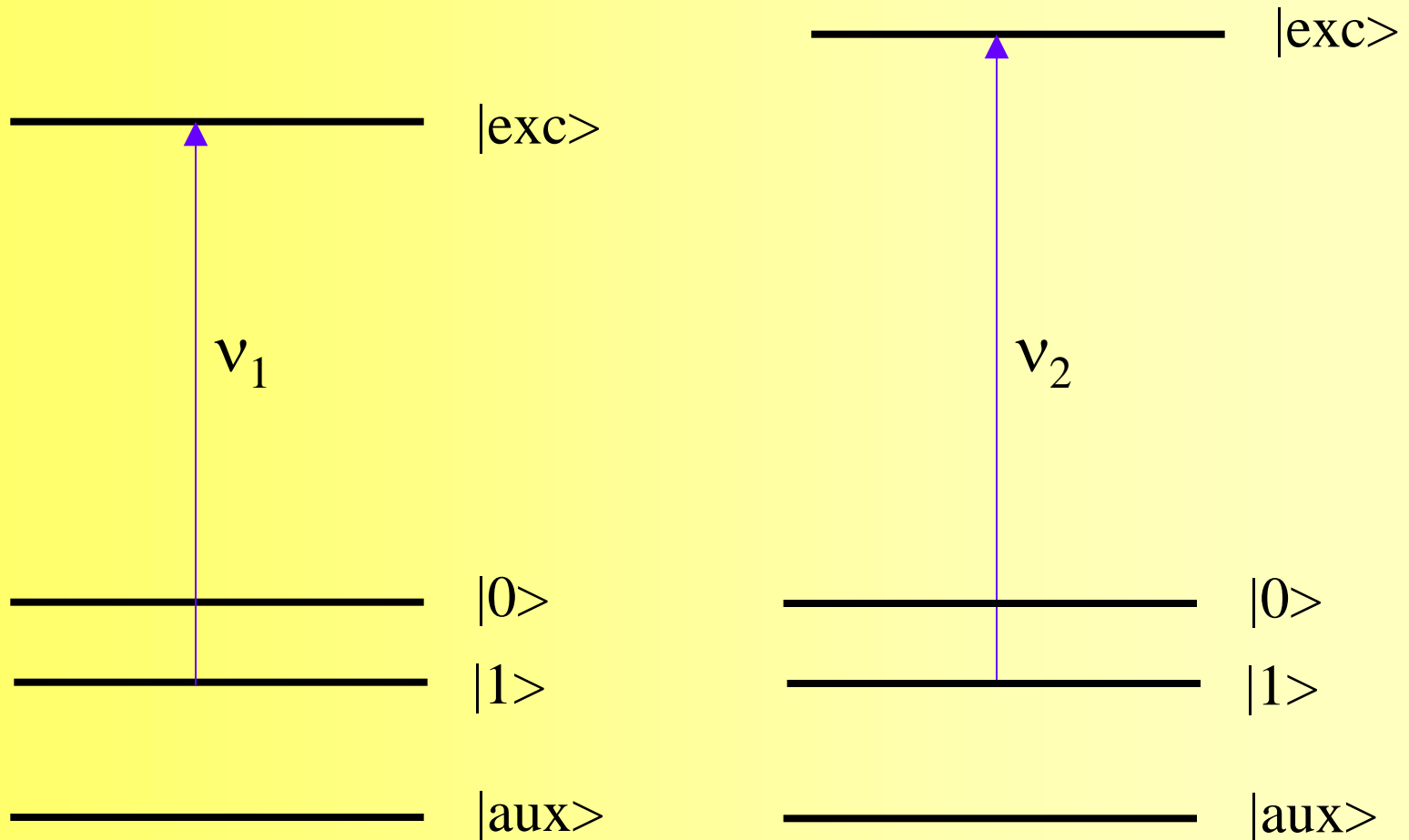
ν_1 ● ν_2 ▲



- Narrow homogeneous line-widths (1-10 kHz)
- Large inhomogeneous line-widths (1-200 GHz)

— $|0\rangle$
— $|1\rangle$

Addressing two different qubits in a rare-earth quantum computer



Requirements for quantum computing

- Coherent two-level systems acting as qubits
- Possibility to manipulate the qubits individually (one-bit gates)
- **Coupling between any two qubits**
(What is a two-bit gate?)
- Possibility for reliable read-out of the individual qubits
- Scalability

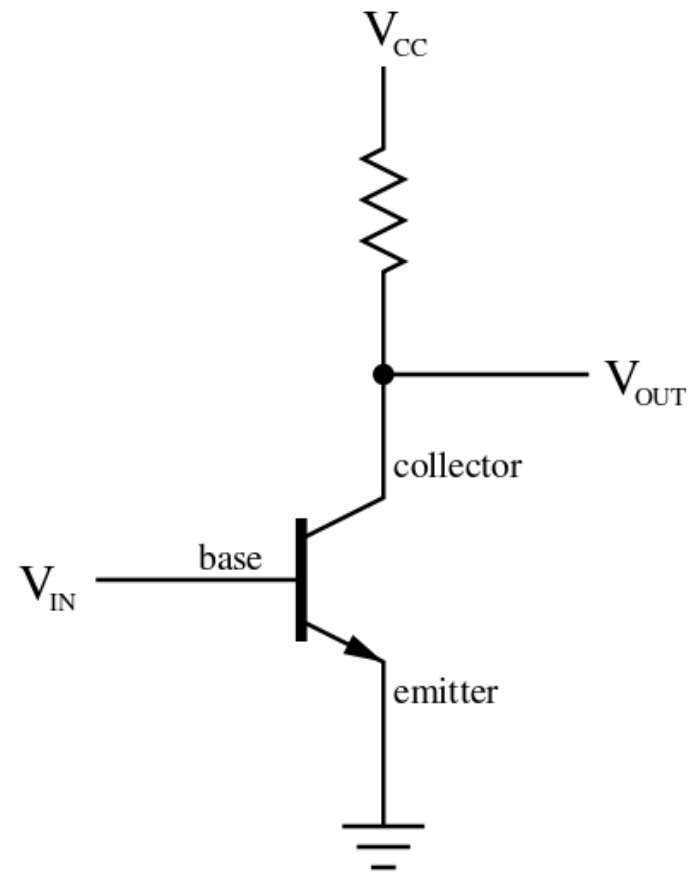


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Gates

Nonlinear elements

**How could one atom control
another atom?**



<http://en.wikipedia.org/wiki/Transistor>



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Universal set of gates

- Arbitrary single qubit operations
- Suitable two-qubit gate, e.g., C-NOT (CONTROL-NOT) (Chapter 13.2.1)

The NOT gate

$$\Psi = \alpha|0\rangle + \beta|1\rangle$$

① $\Psi = \alpha|e\rangle + \beta|1\rangle$

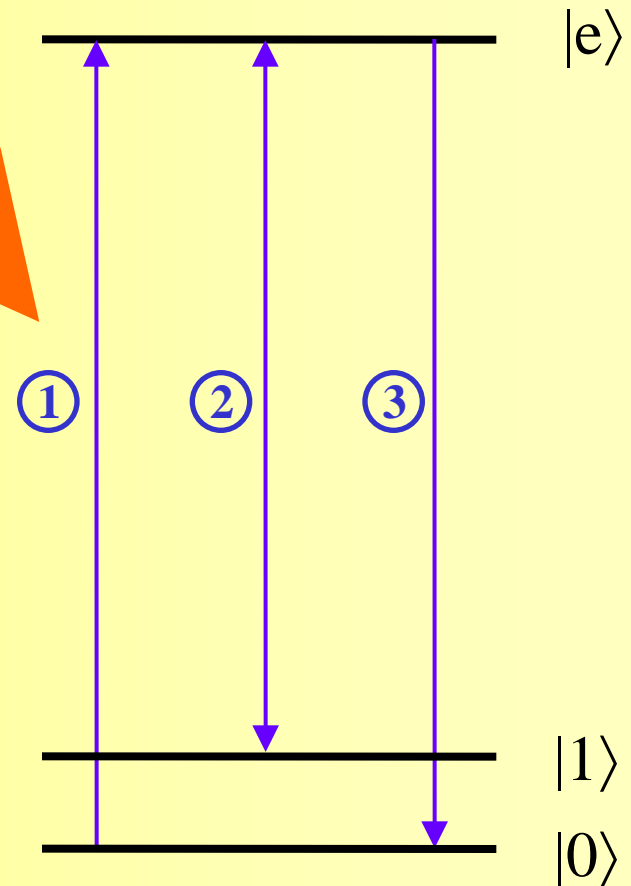
② $\Psi = \alpha|1\rangle + \beta|e\rangle$

③ $\Psi = \alpha|1\rangle + \beta|0\rangle$

Two of the hyperfine levels of the ions are used as the qubit states

Quantum bit

π -pulses




Controlled-NOT quantum gate

Control

v_1 


 $|e\rangle$

Target

v_2 


 $|e\rangle$

Control

v_1 

 $|e\rangle$


Target

v_2 

 $|e\rangle$


No change
to target

 $|1\rangle$

 $|0\rangle$

 $|1\rangle$

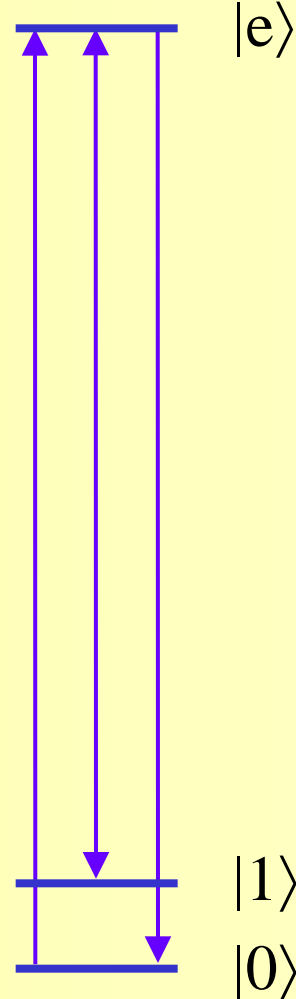
 $|0\rangle$

 $|1\rangle$

 $|0\rangle$

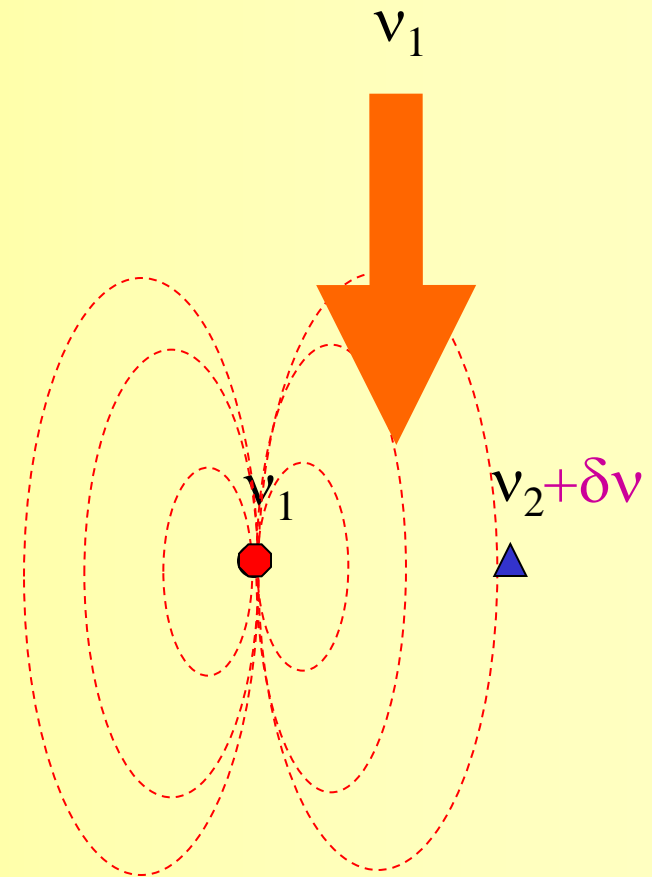
 $|1\rangle$

 $|0\rangle$



Dipole-dipole interaction

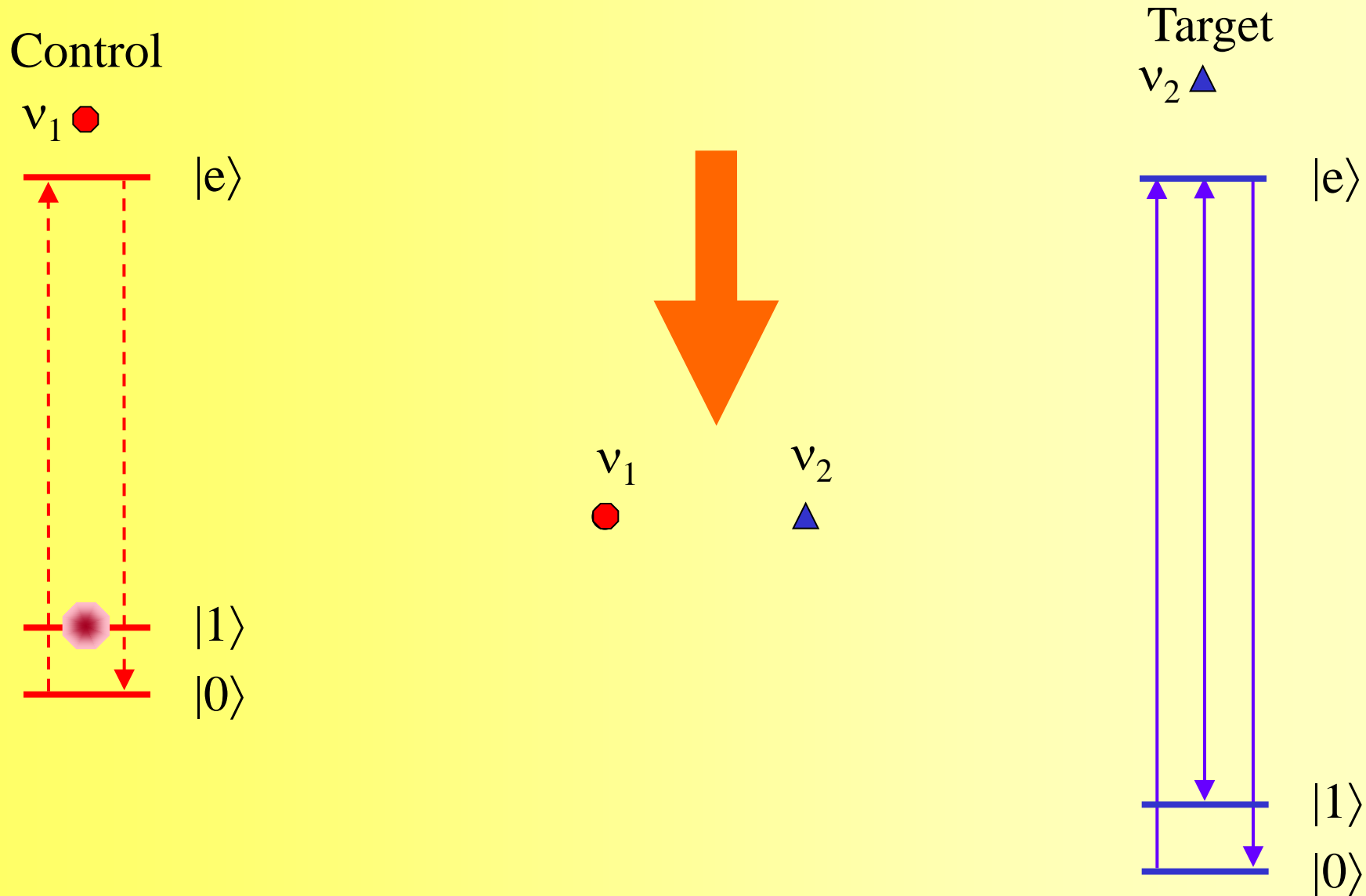
1. Two ions absorbing at different frequencies are located close to each other in the crystal lattice. In a non-centrosymmetric site the ions will have a permanent electric dipole moment
2. One of the ions is excited on its optical transition. The ion has a different permanent dipole moment in its excited state.
3. This change in dipole moment is sensed by the other ion causing its absorption frequency to change.



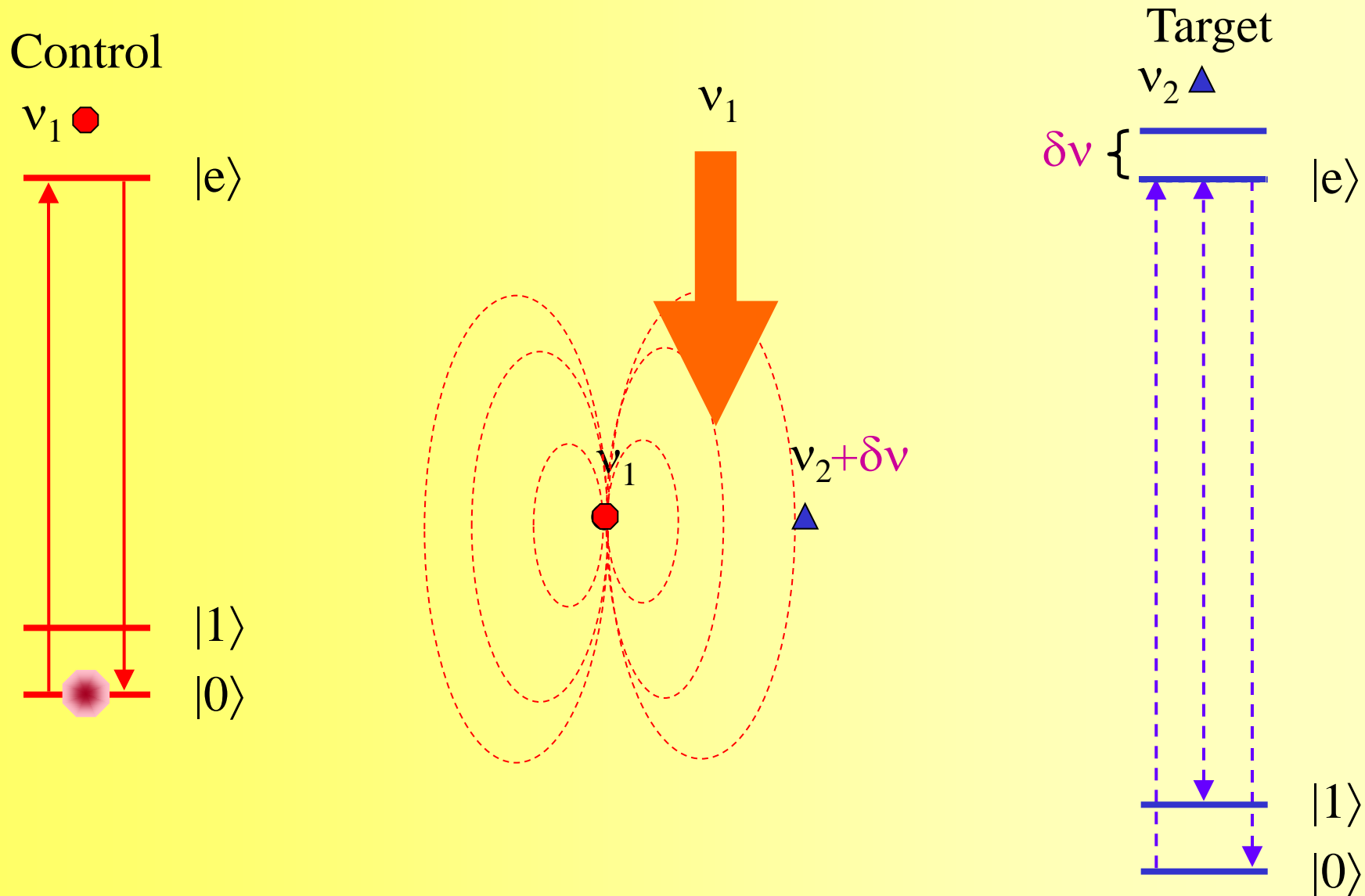
Dipole-dipole interaction strength in rare-earth crystals

- Approximate numbers
- | Ion distance | frequency shift |
|---------------------|------------------------|
| 100 nm | 1 line width |
| 10 nm | 1000 line widths |
| 1 nm | 1000000 line widths |

Controlled-NOT quantum gate



Controlled-NOT quantum gate



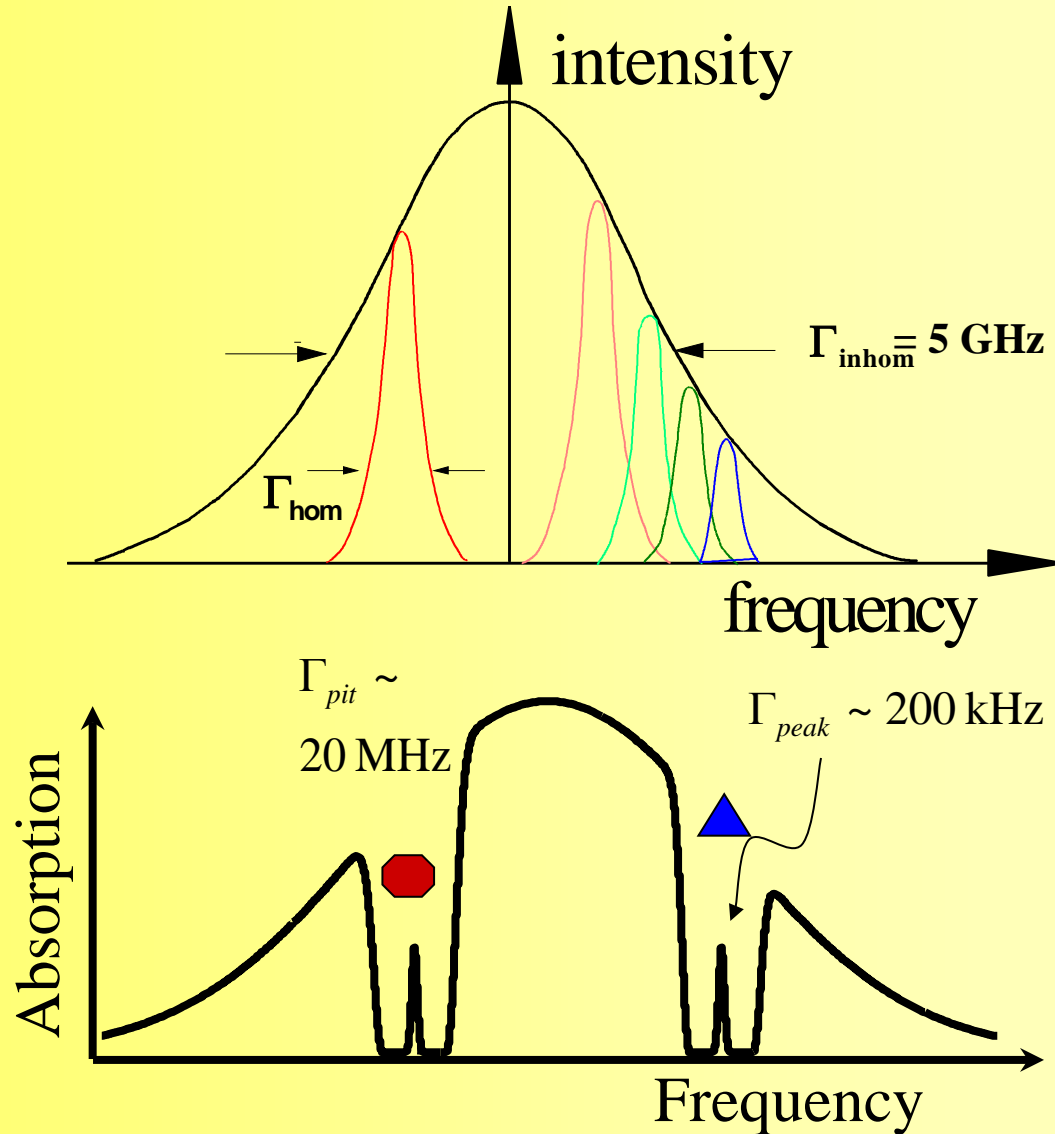
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Qubit creation

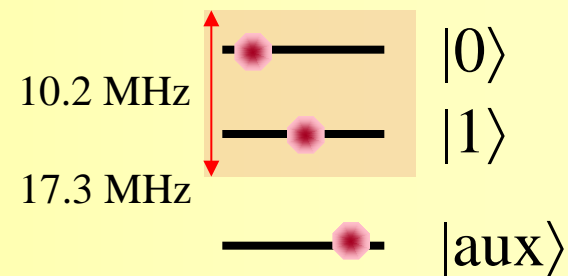
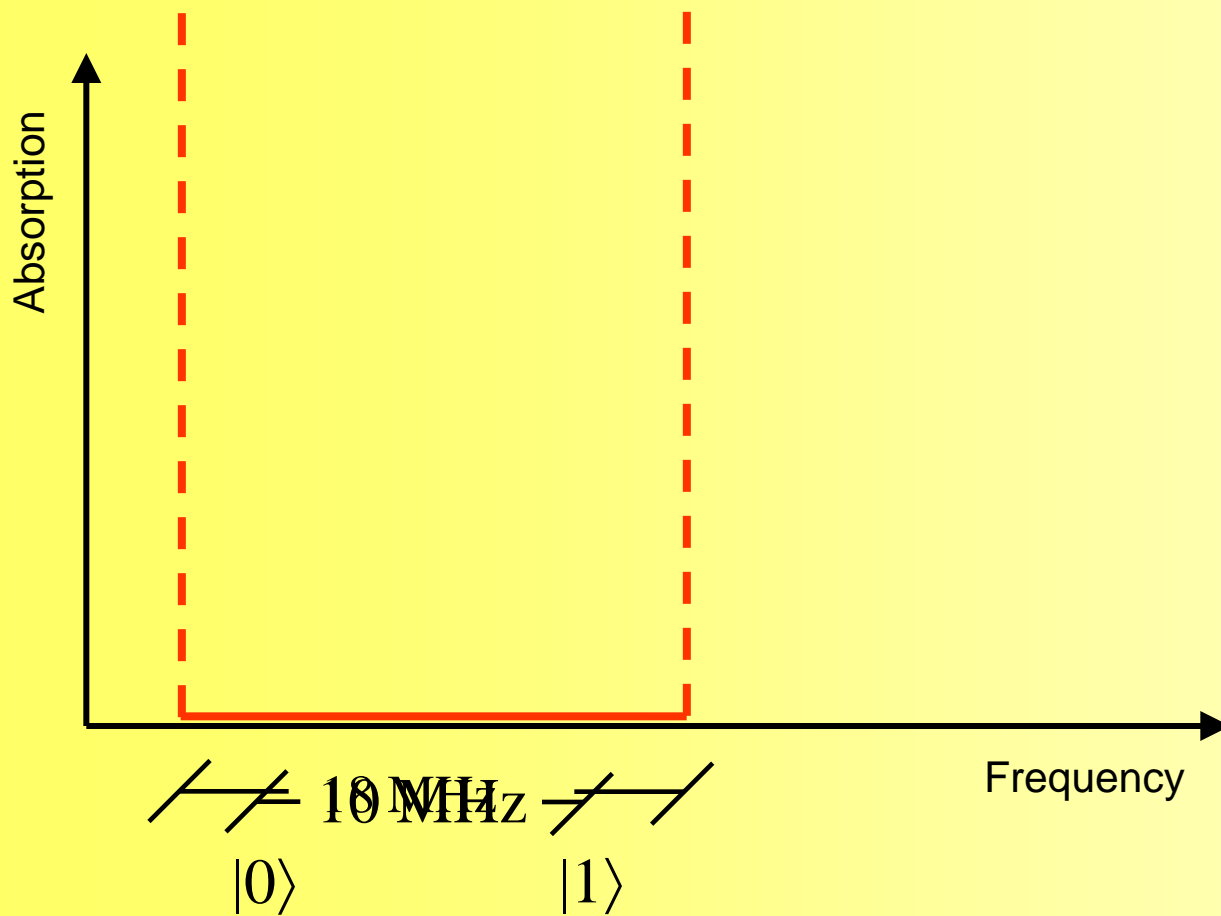
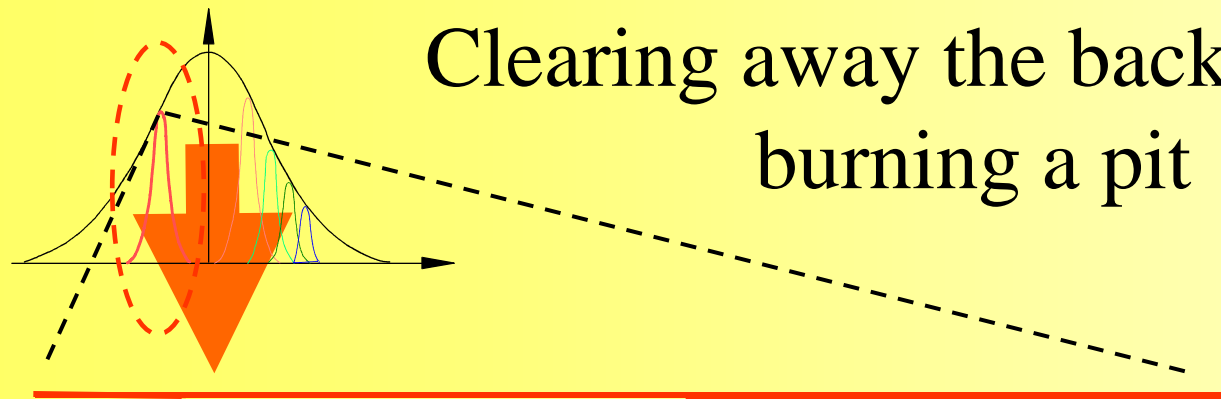
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Clearing away the background - burning a pit

Pr:Y₂SiO₅

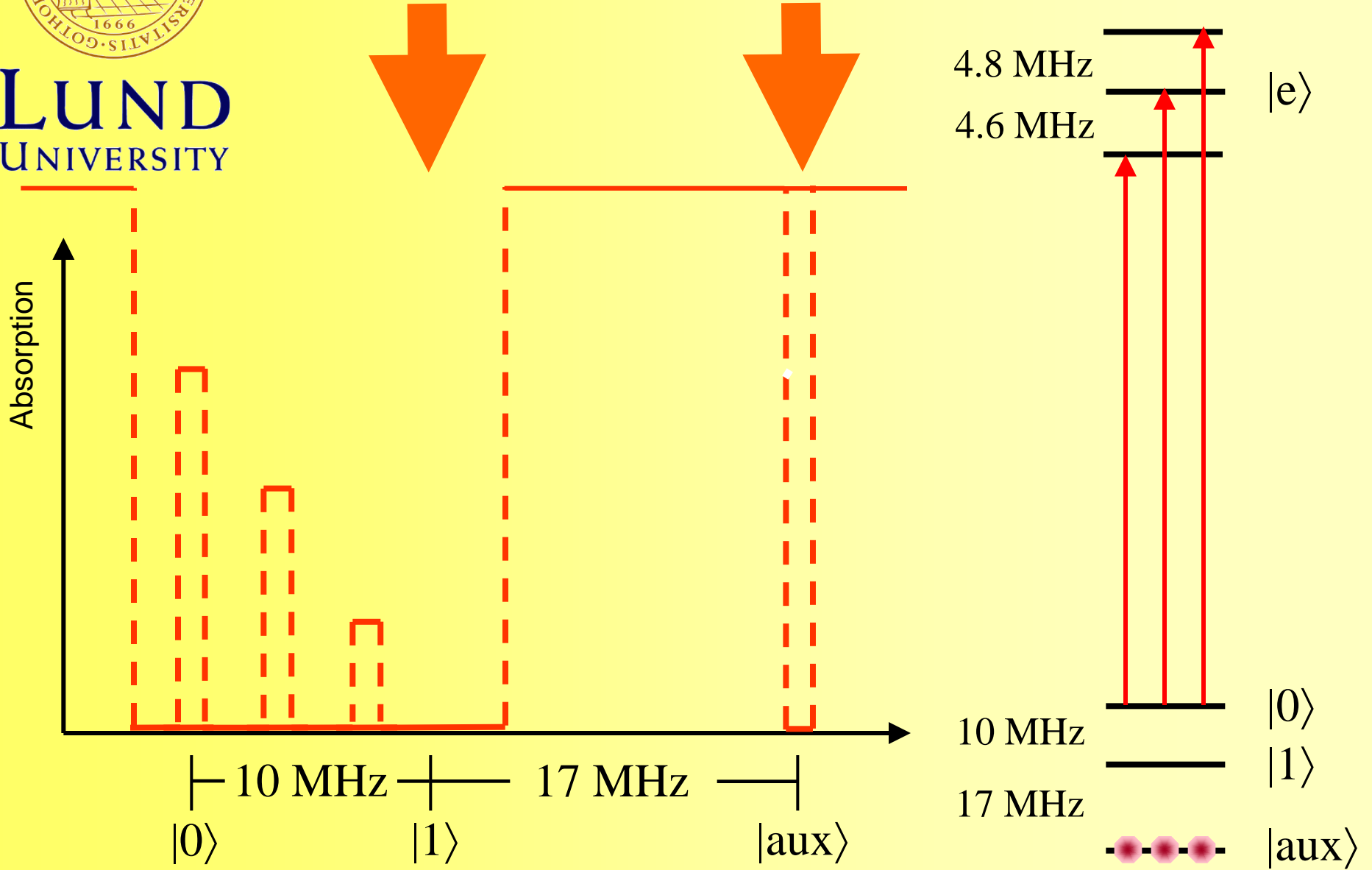
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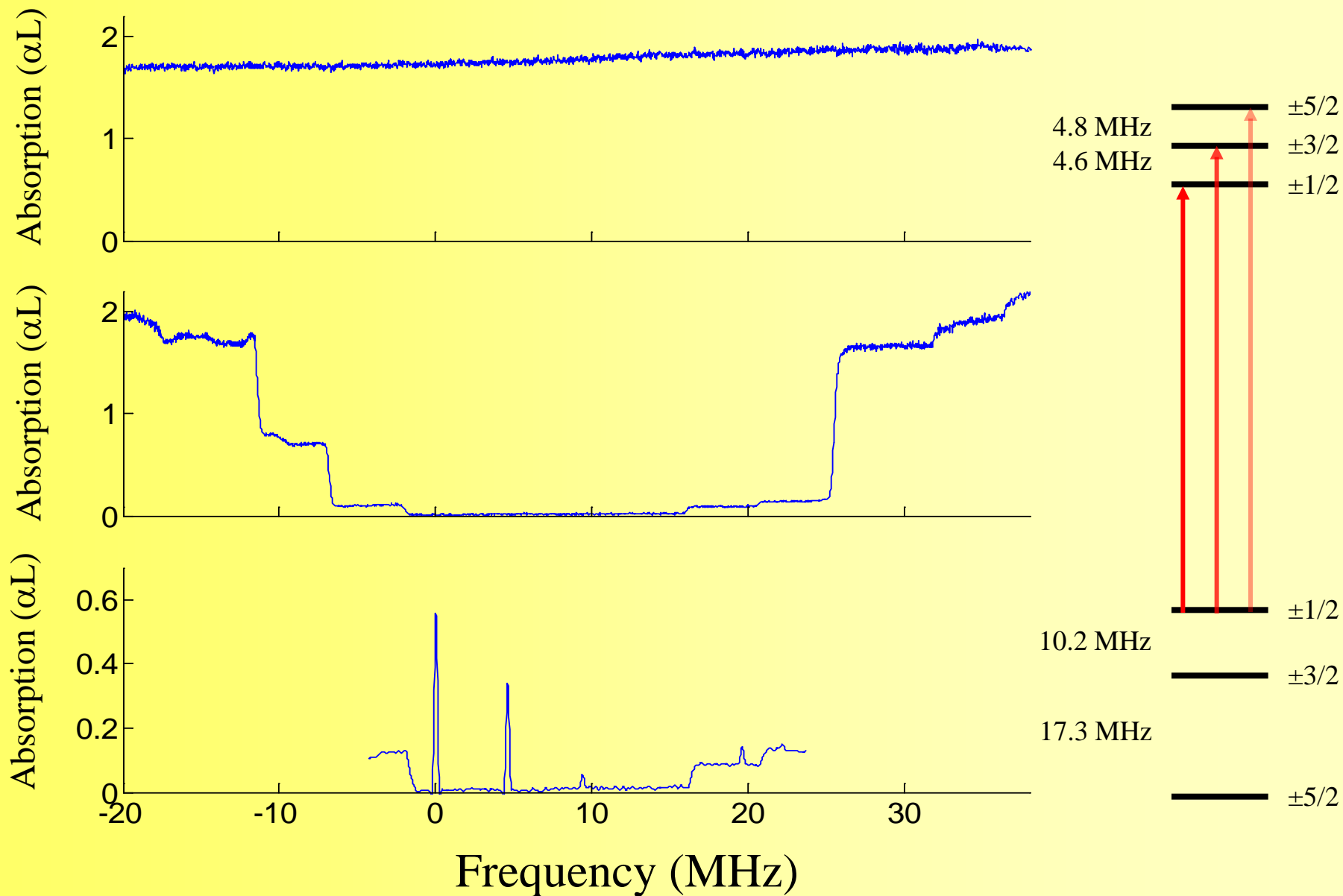


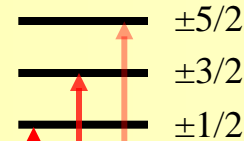
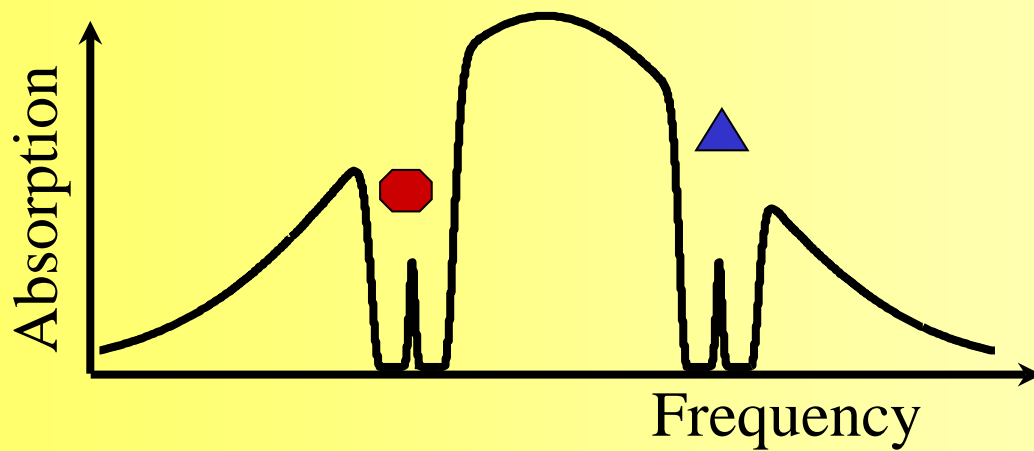
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Burning back a peak

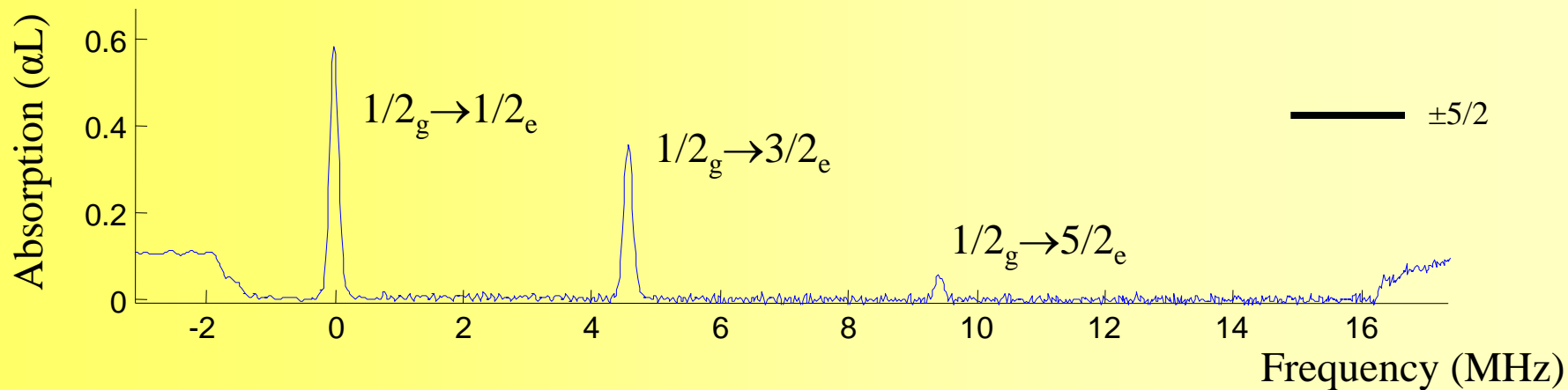
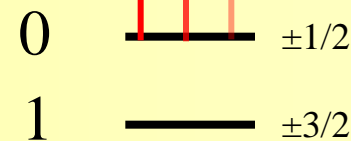


Pit & peak creation





Qubit zoom in





Requirements for quantum computing

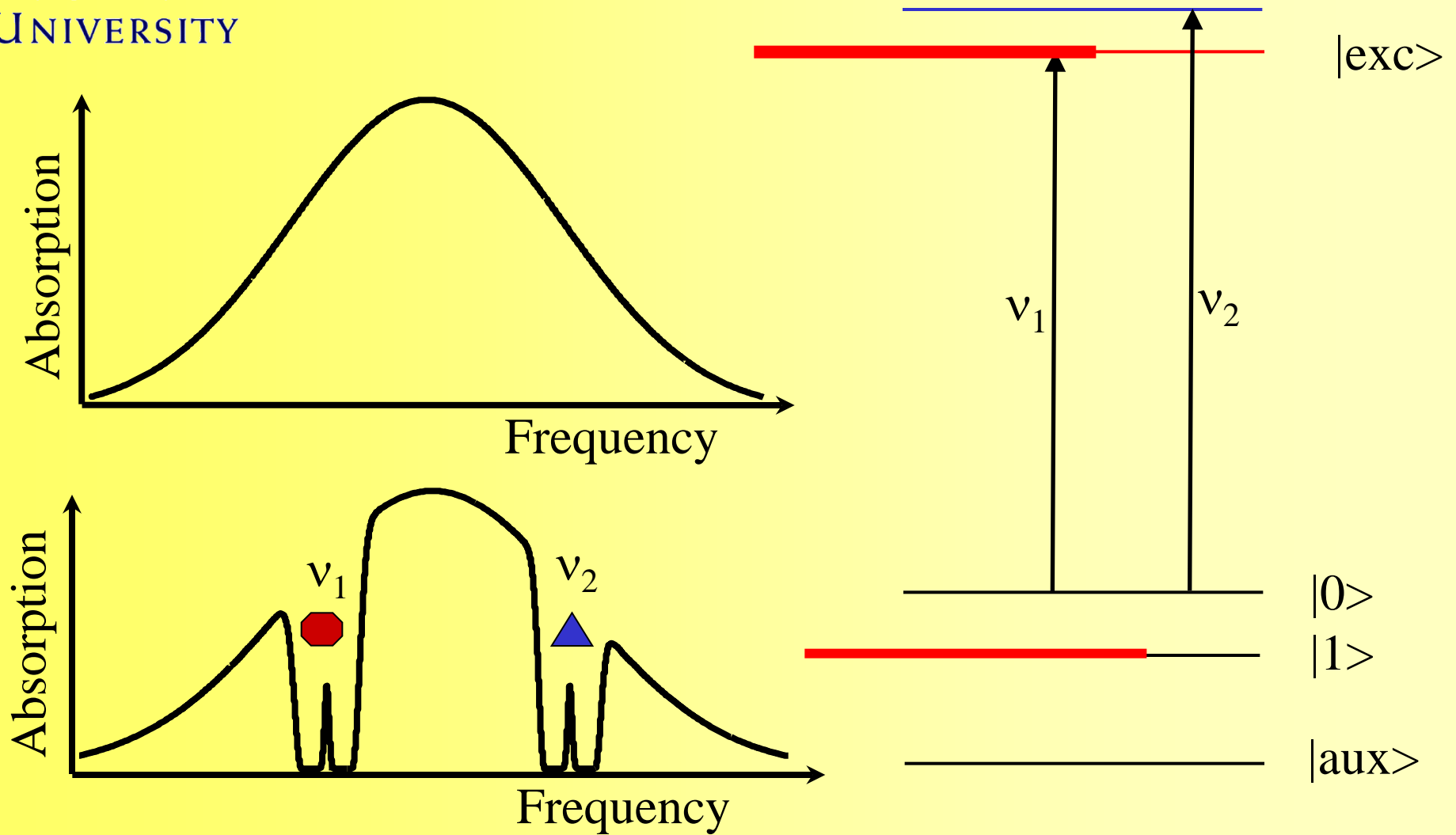
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- Coherent two-level systems acting as qubits
- Possibility to manipulate the qubits individually (*single qubit operations*)
- Coupling between any two qubits (*two-bit gates*)
- Possibility for reliable read-out of the individual qubits
- **Scalability**

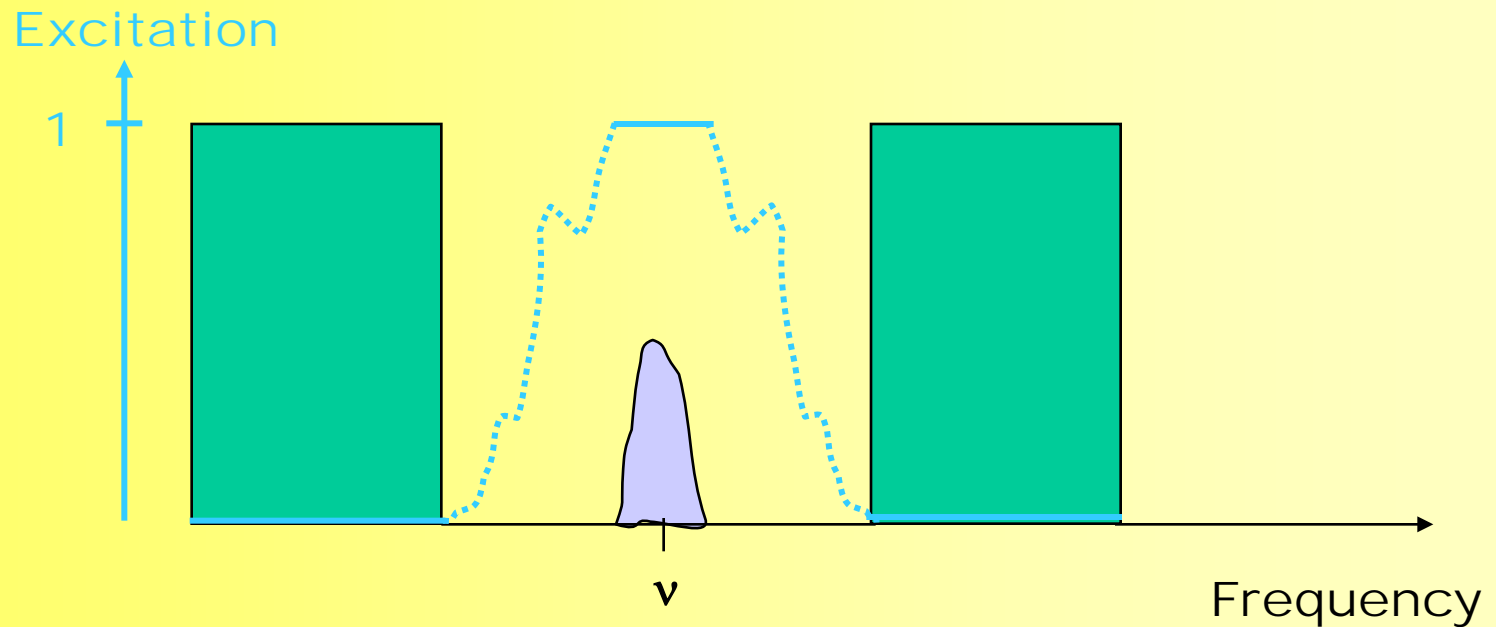


Inhomogeneous absorption profile is tailored to create qubit structures

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How to interact with the qubit ions without interacting with ions at nearby absorption frequencies

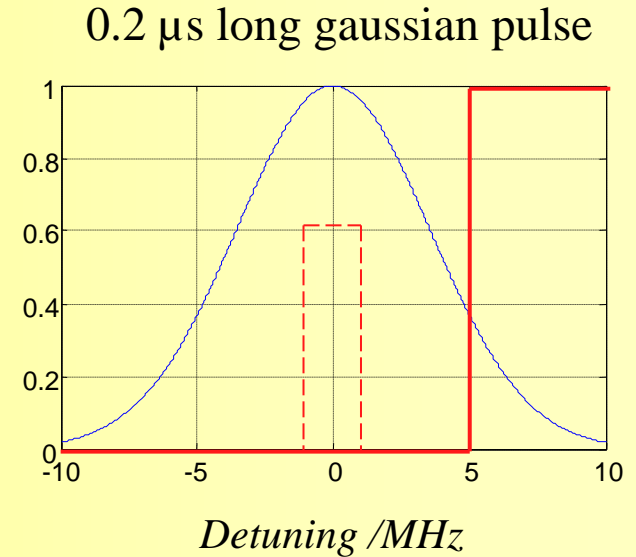
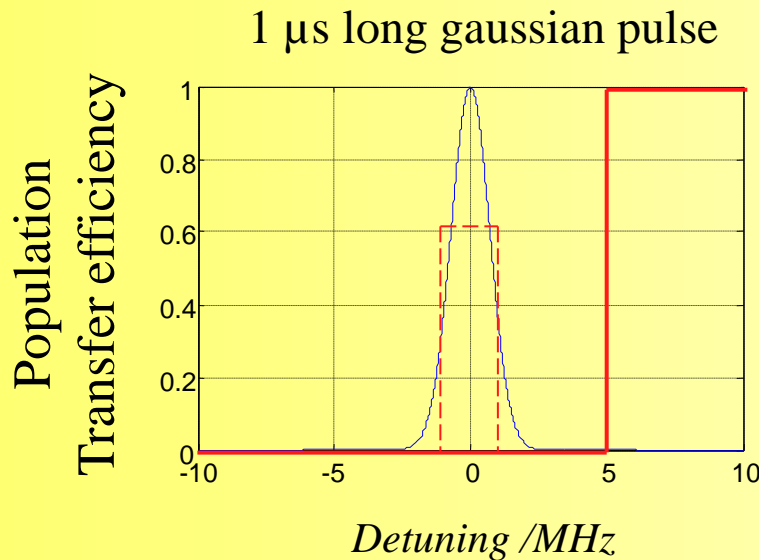
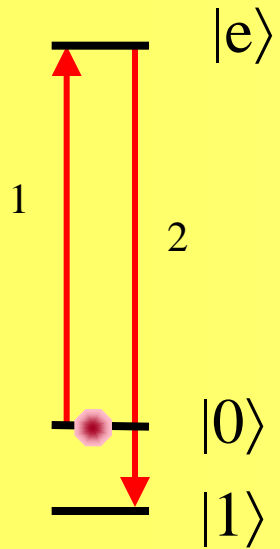




Performing qubit operations

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Consider pulses with gaussian temporal shape



- Problem #1: Not the same Rabi frequency everywhere
- Problem #2: Ions outside the qubit may be excited

Pulse shapes for coherent transfer of population

This work was carried out by Ingela Roos together with Klaus Mølmer

- “Robust quantum computing with composite pulse and coherent population trapping”, Phys Rev A **69**, 22321 (2004)



Requirements

- Complete transfer of the peak of ions
- No excitation of surrounding ions
- Compensate for inhomogenous qubit width

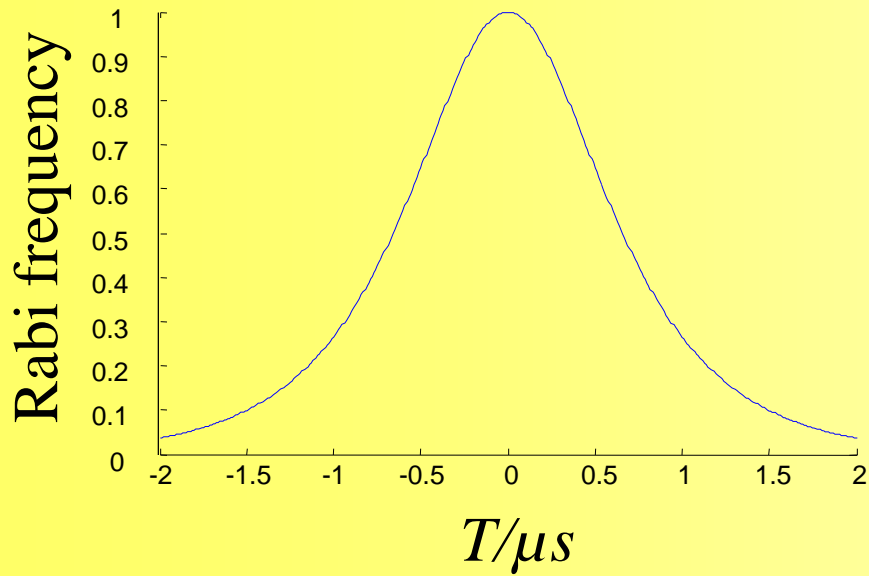


Complex hyperbolic secant pulse

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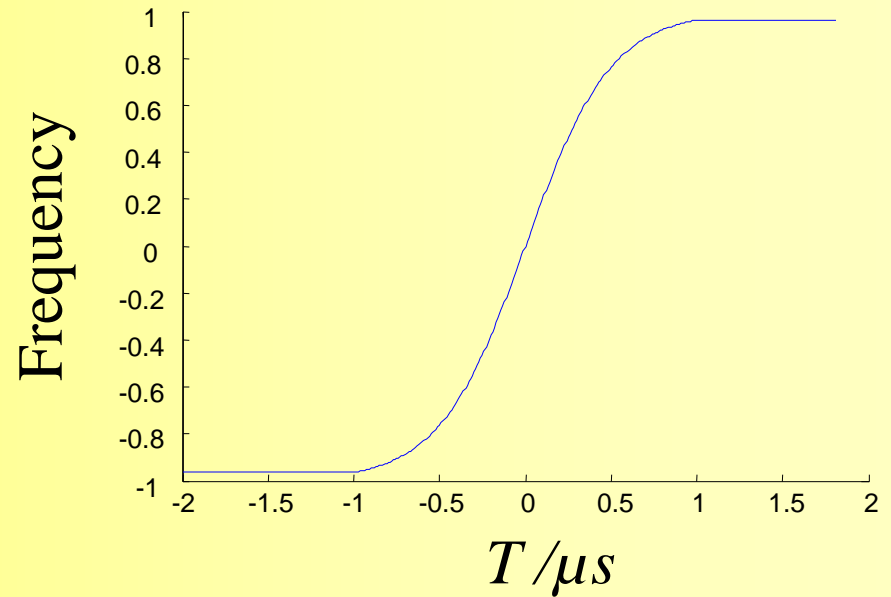
Sechyp

Amplitude envelop



Tanhyp

Frequency chirp



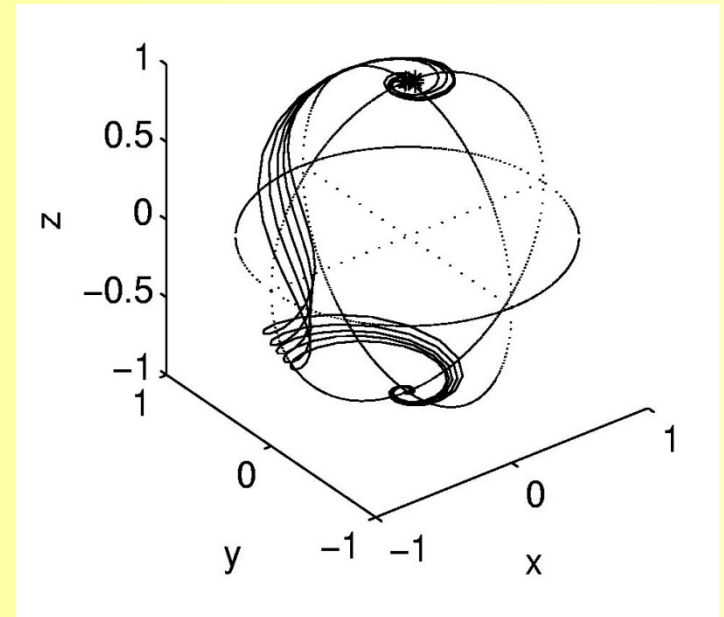
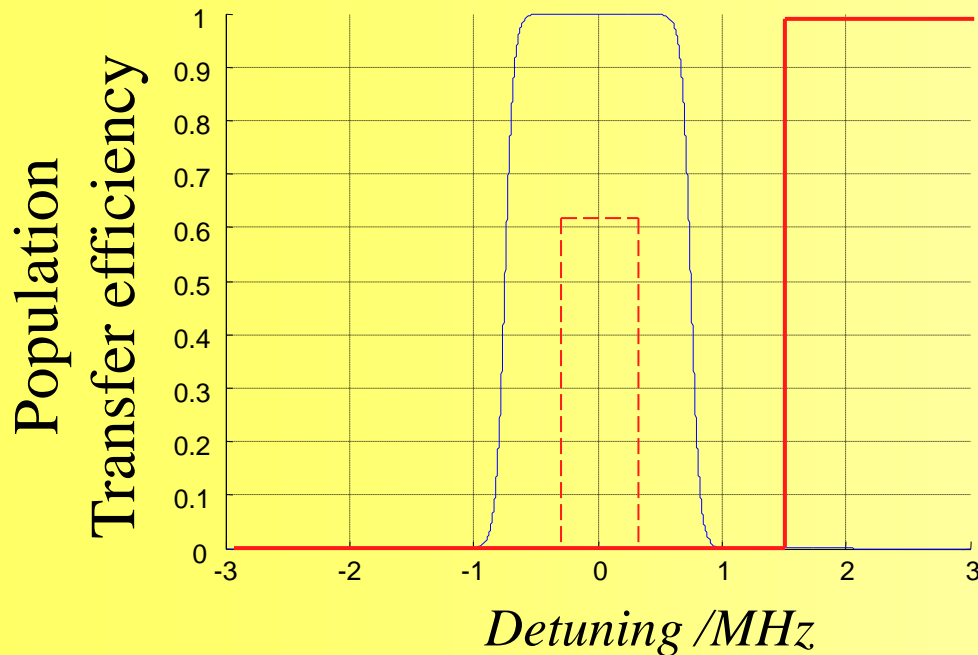


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Complex hyperbolic secant pulse

- + Above a certain threshold intensity the operation is insensitive to differences in Rabi frequency
- + Compensates for dephasing due to inhomogeneous broadening
- Can only handle pole to pole transfers

Evolution on the Bloch sphere



Outline

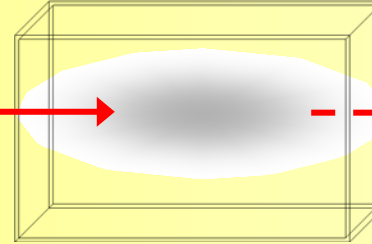
- Quantum computing and quantum resources
- Quantum bits (qubits) & the Bloch sphere
- The quantum control lab and the Lund approach to quantum computing



Quantum memories

Light encoded with quantum information

$|\Psi^{\text{in}}\rangle$

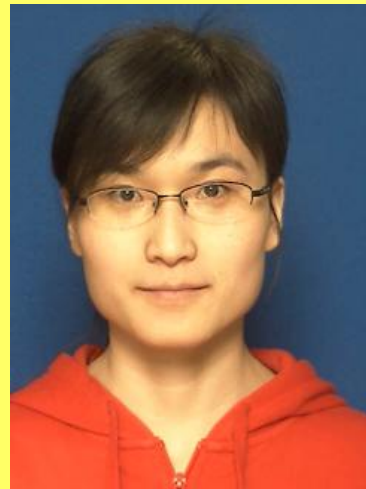
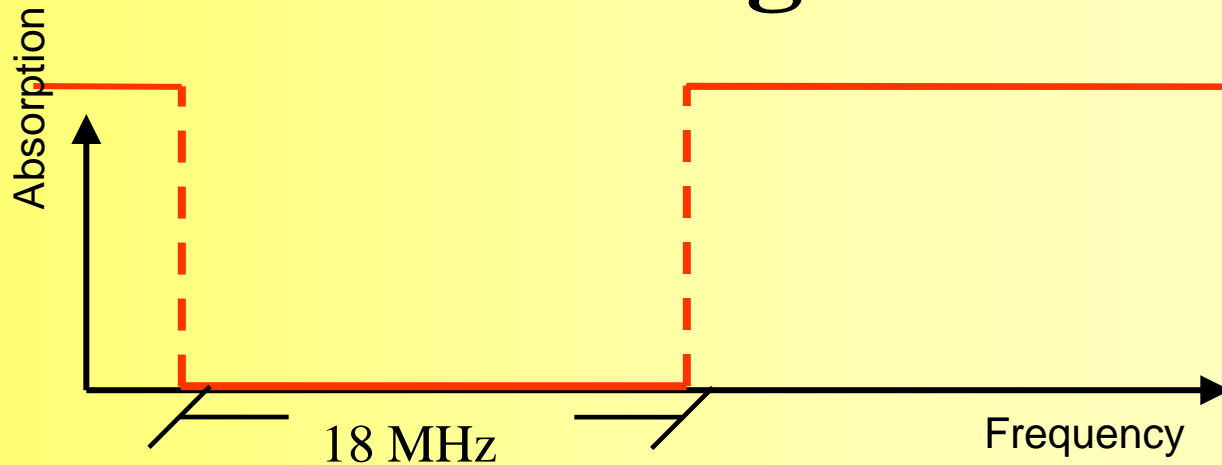


Retrieved signal

$|\Psi^{\text{out}}\rangle$

$$\text{Storage Fidelity} = |\langle \Psi^{\text{in}} | \Psi^{\text{out}} \rangle|^2 \leq 1$$

Slow light



Qian Li



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Quantum computing

Improving operation fidelity

$$\text{Fidelity} = |\langle \Psi^{\text{exp}} | \Psi^{\text{theory}} \rangle|^2 \leq 1$$

Reading out the state of a single ion



Adam Nilsson

Slow light

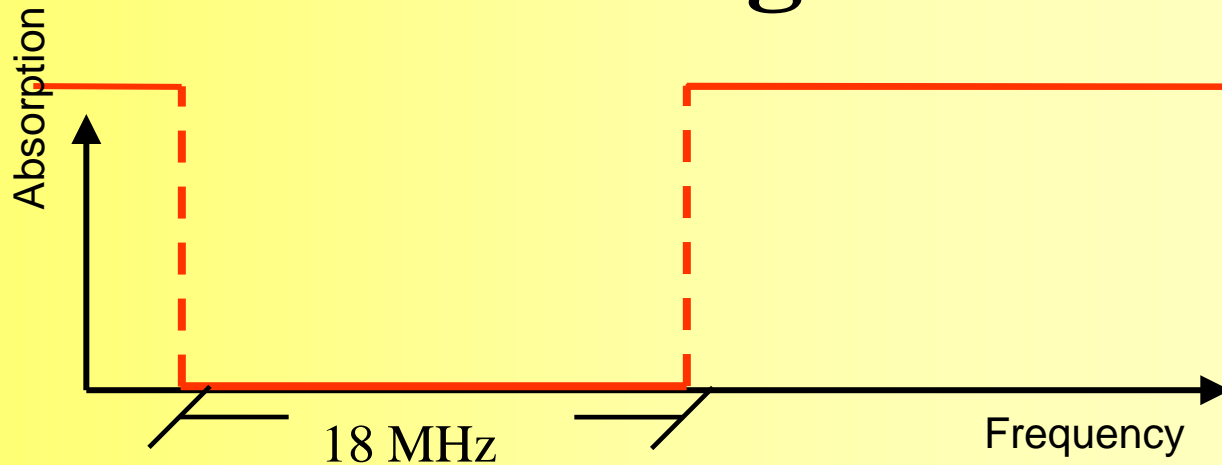




Photo:
Tomas Svensson