Chapter 7: Dielectric layered media

P7-1: The following wave-transfer matrix is given:

\[
M = \frac{1}{2n_2} \begin{bmatrix} n_2 + n_1 & n_2 - n_1 \\ n_2 - n_1 & n_2 + n_1 \end{bmatrix} \times \begin{bmatrix} \exp\{-j\varphi\} & 0 \\ 0 & \exp\{j\varphi\} \end{bmatrix}
\]

What does it represent? Find the corresponding scattering matrix.

P7-2: Derivation of the Airy formulas: We consider the transmission through a cascade of two systems, separated by free space. The wave transfer matrix can be written as:

\[
M = M_{23} \times M_{3} \times M_{12} = \frac{1}{t_{32}} \begin{bmatrix} t_{32}l_{32} - r_{32}r_{32} & r_{32} \\ -r_{32} & 1 \end{bmatrix} \times \begin{bmatrix} e^{-j\varphi} & 0 \\ 0 & e^{j\varphi} \end{bmatrix} \times \frac{1}{t_{21}} \begin{bmatrix} t_{21}l_{21} - r_{21}l_{21} & r_{21} \\ -r_{21} & 1 \end{bmatrix}
\]

We can find the Airy formulas for the transmission and reflection from the corresponding scattering matrix. The expression for the transmission is a bit lengthy, so let’s just derive the one for reflection.

P7-3: We make an anti-reflection coating to minimize the Fresnel back reflection for an incoming wave at plane incidence at a boundary between two dielectric materials with refractive indices \(n_1\) and \(n_3\), respectively. We realize the anti-reflection coating by introducing a thin dielectric film of thickness \(\lambda/4\) and refractive index \(n_2\) in between (see figure).

a) Find the wave transfer matrices and calculate the wave transfer matrix for the system.
b) Find the scattering matrix for the system (no need to write it out).
c) Find the refractive index \(n_2\) for which the back-reflection vanishes.

P7-4: A mirror Fabry-Perot etalon is made of two lossless partially reflective mirrors (see section 7.1B in FoP). The maximum power transmission is \(\tau_{\text{max}}=1\), the thickness is \(d=500 \, \mu \text{m}\) and the width of each transmission resonance is \(\delta_\nu=600 \, \text{MHz}\). Calculate the Finesse of the etalon and find the absolute of the amplitude reflectances of two mirrors that make the etalon.

P7-5: Mirror and slab Fabry-Perot etalon: Use some algebra software you have access to (like Matlab, Maple, Mathematica, etc.) and plot the power transmission and reflection of a mirror and a dielectric slab Fabry-Perot etalon vs. \(\varphi\). Use the Airy formulas of transmission and reflection (7.1-8 FoP).

- Use the following parameters for the mirror etalon: \(R_1=R_2=0.5\) (power reflections). Remember that \(\arg\{r_1r_2\}=0\) for a mirror etalon.
- Use the following parameters for the slab etalon: \(n_1=n_3=1, n_2=3.5\)
- Test if the two etalons are reciprocal by comparing the phase shifts for the transmitted and reflected waves (FoP 7.1-13).
Answers chapter 7:

P7-1:

\[
M = \frac{1}{2n_2} \begin{bmatrix} (n_2 + n_1) e^{-j\phi} & (n_2 - n_1) e^{j\phi} \\ (n_2 - n_1) e^{-j\phi} & (n_2 + n_1) e^{j\phi} \end{bmatrix}
\]

\[
S = \begin{bmatrix} t_{12} & r_{21} \\ r_{12} & t_{21} \end{bmatrix} = \frac{1}{D} \begin{bmatrix} AD - BC & B \\ -C & 1 \end{bmatrix} = \frac{1}{(n_2 + n_1) e^{j\phi}} \begin{bmatrix} 4n_2n_1 & (n_2 + n_1) e^{j\phi} \\ -(n_2 - n_1) e^{-j\phi} & 2n_2 \end{bmatrix}
\]

P7-3: \( n_2 = \sqrt{n_1n_3} \)

P7-4: Finesse \( F = 500 \), \(|r| \approx 0.997\)