Fermat’s principle: Light travels along the paths such that the optical path length is an extremum. In general, light travels along the path of least time.

\[ \tau = \frac{nd}{c_0} \]
\[ \frac{1}{z_1} + \frac{1}{z_2} = -\frac{2}{R} \]

\[ M = \frac{y_2}{y_1} = -\frac{z_2}{z_1} \]
$n_1 < n_2$

External refraction

$n_1 > n_2$

Internal refraction
Refraction at a spherical boundary

\[
\frac{n_1}{z_1} + \frac{n_2}{z_2} = \frac{n_2 - n_1}{R}
\]

\[
M = \frac{y_2}{y_1} = -\frac{n_1 z_2}{n_2 z_1}
\]
\[
\frac{1}{z_1} + \frac{1}{z_2} = -\frac{2}{R}
\]

\[
M = \frac{y_2}{y_1} = -\frac{z_2}{z_1}
\]
$n_1 > n_2$
Use of glass fibers in the 1950’s

Decoration
Medicine (gastroscope)
Image transmission

Too low transmission for communication!

Attenuation:

$$\alpha = \frac{10}{L} \log_{10} \left( \frac{P(0)}{P(L)} \right)$$

1000 dB/km
After 20 m, 1% light left
After 1 km, no light left
Transmission in an Optical fiber

Losses due to absorption, scattering, propagation, bending etc..

IMPURITIES (Iron)

Kao and Hockham 1966

![Graph showing attenuation coefficient vs. wavelength with peaks and dips indicating different types of losses: Rayleigh scattering, Infrared absorption, OH absorption.]
Low-loss optical fiber

1970  16 dB/km - 4 dB/km

After 1 km  40 % light left

Now  < 0.2 dB/km

After 1 km  95% light left

Corning Glass Works
Fused Silica – Doped with titanium for the core
GRaded INdex Optics

\[
\frac{d}{ds} \left( n \frac{d\vec{r}}{ds} \right) = \nabla n
\]
Ray equation

\[
\begin{cases}
\frac{d}{dz} \left( n \frac{dx}{dz} \right) = \frac{\partial n}{\partial x} \\
\frac{d}{dz} \left( n \frac{dy}{dz} \right) = \frac{\partial n}{\partial y}
\end{cases}
\]
Paraxial ray equation

Ex: \( n^2(y) = n_0^2 \left( 1 - \alpha^2 y^2 \right) \)
\[ \frac{n_1}{z_1} + \frac{n_2}{z_2} = \frac{n_2 - n_1}{R} \]

Convex: \( R > 0 \); concave: \( R < 0 \)
Convex: $f > 0$; concave: $f < 0$

Concave: $R < 0$; convex: $R > 0$

\[ \frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f} \]

\[ \frac{1}{z_1} + \frac{1}{z_2} = -\frac{2}{R} \]
\[ M = M_N \cdots M_2 M_1. \]