Polarized Light

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Introduction

In this lab you will work with several important concepts in optics. First you will study different optical components that create and change the polarization state of light. We create linearly polarized light with filters, built on the principle of selective absorption (dichroism). Linearly polarized light may then be changed into an arbitrary state using a wave retarder, built on the principle of birefringence in an anisotropic media. The plane of vibration in a linearly polarized beam may also be rotated using optically active media in both solid and liquid form. During the lab you will also learn how to work with a modern grating spectrometer equipped with a reflection grating and a CCD camera.

By measuring the optical properties with good spectral resolution you will be able to determine the wavelength dependence of the refractive index, i.e. the dispersion of the media. This phenomenon explains for example the beautiful colors seen in the rainbow or refracted from a diamond ring.

The description and production of polarized light is discussed in detail in chapter 6 of the textbook Photonics. Dispersion is treated in chapter 5.5 and the diffraction grating is briefly mentioned in chapter 2.4. Some aspects of polarized light of particular relevance for this laboratory exercise are also included in Appendix 1 and 2 of this instruction manual. The Czerny-Turner spectrometer that we will use should be quite familiar to you from the optical design project using FRED.
Preparatory problems

1. Which state of polarization is described by the following expressions of the electric field:
   a) \( \vec{E} = E_0 \cdot (\vec{e}_x \cdot \sin(kz - \omega t) + \vec{e}_y \cdot \cos(kz - \omega t)) \)
   b) \( \vec{E} = 5 \cdot \vec{e}_x \cdot \sin(kz - \omega t + \pi / 2) + 3 \cdot \vec{e}_y \cdot \sin(kz - \omega t) \)

2. The transmitted intensity of linear light through an ideal linear polarizer is given by the law of Malus. Formulate this law.

3. What does it mean that a material is birefringent?

4. What happens if unpolarized light passes through a birefringent crystal perpendicular to the optics axis?

5. What does it mean that a material is optically active?

6. Linearly polarized green and yellow light passes through an optically active medium. Which color is rotated the most?

7. White light strikes a plane reflection grating with 300 lines/mm at an angle of 15° to the normal. At which angles do you observe 550 nm light in the first spectral order?
Experiments

1. The spectrometer

The optical layout of the spectrometer, from Acton Research Corporation, is shown in Figure 1. In our set-up the exit slit assembly has been removed to allow a direct visual inspection of the spectrum through a lens. The spectrometer and the CCD camera are controlled by the software WinSpec, which is briefly described in Appendix 3.

![Spectrometer Diagram]

Figure 1. Czerny-Turner spectrometer SpectraPro 300i

- Place a Cd lamp in front of the entrance slit. Remove the lid of the spectrometer and study the construction. Compare with your previous ray-tracing exercise using FRED. Take a piece of white paper and follow the light path through the system. Be careful not to touch the mirrors or the grating
- Use the WinSpec program to select the 300 l/mm grating and a centrum wavelength of 560 nm and study the spectrum by eye at the exit slit. Compare with Figure 2.

<table>
<thead>
<tr>
<th>Color</th>
<th>Wavelength (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Violet</td>
<td>467.81</td>
</tr>
<tr>
<td>Blue</td>
<td>479.99</td>
</tr>
<tr>
<td>Green</td>
<td>508.58</td>
</tr>
<tr>
<td>Red</td>
<td>643.85</td>
</tr>
</tbody>
</table>

Figure 2. Cd-spectrum with wavelengths in units of 1 nm.
• Register the same Cd spectrum quantitatively using the CCD camera. Select a suitable combination of paper towels in front of the slit and exposure times so that you obtain a spectrum with a maximum intensity close but not exceeding 65000 counts (maximum with 16-bits) in under 1 s.

• Compare the peak values with the once given above. If they diverge calibrate the spectrum by, in the calibration setup, assigning the correct wavelength to the pixel value of the peaks.

2. **Birefringence - wave retarder.**

• Start with recognizing what is in front of you. For the rest of the lab you will use a lamp as a white-light source. Study the spectrum of this lamp and note the big difference between a line spectrum from a low pressure spectral lamp and the continuous, black-body spectrum from a heated filament. Find the polarizing filters. In between there is a holder for the crystals (figure A2-2).

2.1 Qualitative investigations

• Use 3 quartz crystals of different thicknesses, cut with the optic axis parallel to the end surfaces, i.e. the light travels perpendicularly to the axis. Put each of the crystals between the crossed polarizers and view the spectrum by eye. Rotate the crystals in its holder and verify that there are several orientations where no light passes through the system. Study how the spectrum changes as the last polarizer is rotated. Explain the results (see appendix 1 and 2).

2.2 Determine $\Delta n = n_e - n_o$ in a direction perpendicular to the optic axis in quartz

Register the spectrum with the last crystal using the CCD. According to Appendix 2 (Eq. A2.2) there will be a maximum intensity in the spectrum whenever the phase difference $\delta = (2m+1)\pi$, i.e. when:

$$d \cdot \Delta n(\lambda) = (2m + 1) \cdot \frac{\lambda}{2}$$

(1)

Since the difference in refractive index $\Delta n$ is almost constant from one maximum to the next we know that $m$ must decrease by one for each new maximum when we move towards higher wavelengths. To calculate the refractive index as a function of wavelength with very high accuracy we could use a *Sellmeier equation* with empirically determined parameters. This is discussed in chapter 5.5 of the textbook. However, a much simpler formula, of sufficient accuracy for our purposes, is that of Cauchy:

$$n = A + \frac{B}{\lambda^2}$$

(2)

Here the constants $A$ and $B$ are slightly different for the ordinary and extraordinary rays, hence $\Delta n = n_e - n_o$ is also given by the same type of equation. If you combine equation 1 and 2 you may determine $\Delta n(\lambda)$ for quartz in a direction perpendicular to the optic axis from the measured spectrum.

Choose 3 of the observed maxima with integer differences $p$ and $q$, as in Figure 3. Determine the wavelengths ($\lambda_1 < \lambda_2 < \lambda_3$) of the maxima.
Figure 3. At the wavelengths \( \lambda_2 \) and \( \lambda_3 \) the value of \( m \) is reduced by \( p \) and \( q \), respectively, relative to the peak at \( \lambda_1 \). In the Figure \( p = 9 \) and \( q = 16 \).

From the 3 wavelengths you obtain the following system of equations

\[
\begin{align*}
\frac{d \cdot (A + \frac{B}{\lambda_1^2})}{2} &= (2m + 1) \frac{\lambda_1}{2} \\
\frac{d \cdot (A + \frac{B}{\lambda_2^2})}{2} &= (2m - p + 1) \frac{\lambda_2}{2} \\
\frac{d \cdot (A + \frac{B}{\lambda_3^2})}{2} &= (2m - q + 1) \frac{\lambda_3}{2}
\end{align*}
\]

From which you may determine \( m \) and the constants \( A \) and \( B \) in Cauchy’s formula for the difference \( \Delta n \).

Draw a diagram of \( \Delta n \) as a function of wavelength!
3 Optical activity

Use a new quartz crystal cut so that light propagate along the optical axis. Since quartz is optically active, light in this direction is transmitted as 2 circularly polarized waves with slightly different index of refraction, $n_R$ and $n_L$, see Figure A2-1a.

3.1 Qualitative investigations

- Do a similar analysis as with the other crystals changing the rotation of the crystal and rotating the last polarizer. How is it different from the other crystals? Explain why. Compare with the predictions of equation A2.3.

- Look through the spectrometer and make sure that you can block any color you want by rotating the analyzer. What is the color on the paper when you block a given color?

3.2 Determine if a quartz crystal is dextero- or levo-rotatory.

Look through the spectrometer and rotate the analyzer. Note if the absorption feature (black line) moves towards the red or the blue part of the spectrum when you rotate the analyzer clock-wise. Combine this observation with the fact that the angle of rotation should be larger for blue than for red light (Eq. A2.4) to determine if your crystal is dextero- or levo-rotatory.

Figure 4. Rotation of red and blue light with a dextero-rotatory crystal. Red light is blocked with the analyser in the new position.

3.3 Determine $\Delta n = n_R - n_L$ in a direction parallel to the optic axis in quartz.

- Set the analyzer to zero degree and rotate the polarizer until they are perpendicular. Put in the quartz crystal.

- Register the spectrum using the CCD-camera and rotate the analyzer to see the absorption feature move in wavelength. Record the minimum of the absorbed feature as a function of angle of the analyzer with steps of 5 degrees.

- Use this data to calculate the angle of rotation as a function of wavelength. Note that depending on which way you rotate the analyzer you measure either the rotation angle $\beta$ or $180 - \beta$. Furthermore, Eq. A2.3 shows that the actual rotation may be
\( \beta_{\text{obs}} + m\cdot180 \) if the crystal is thick enough. Use the fact that the rotatory power of quartz around 590 nm should be about 20 °/mm and that the rotation should be larger for blue than for red light to find the proper solution.

- Use Eq. A2.3 and the observed angle of rotation for one wavelength to determine \( \Delta n = n_R - n_L \). Compare with the value you got above for light propagating perpendicular to the optic axis.

- Finally, determine the rotatory power as a function of wavelength and plot the results.
Appendix 1 Description of polarized light

In chapter 6 of the textbook Photonics, polarized light is described by using the Jones vectors. Here we use a more explicit way of describing the light as a superposition of 2 orthogonal oscillations with a relative phase of $\delta$. Thus, the electric field of a polarized wave propagating along the z-axis may be written

$$\vec{E}(x, y, z, t) = E_{0x} \cdot \vec{e}_x \cdot \sin(kz - \omega t + \delta) + E_{0y} \cdot \vec{e}_y \cdot \sin(kz - \omega t) \tag{A1.1}$$

In general the light is elliptically polarized as shown in Figure A1-1, for the special case of $E_{0x} = E_{0y}$.

![Figure A1-1. State of polarization for different relative phases $\delta$ and with $E_{0x} = E_{0y}$.](image)

When the 2 components have a relative phase of $\delta = m\pi$ the light is said to be linearly polarized. Another special case occurs for $\delta = (2m + 1)\pi/2$ and $E_{0x} = E_{0y}$ when the light is circularly polarized. See also Figures 6.1-3 and 6.1-4 in "Photonics".

It is important to note that linearly polarized light may be written as a superposition of 2 circularly polarized waves. Using the notation R(right) and L(left) we find from Eq. A1.1 with $\delta = \pi/2$:

$$\vec{E}_L = E_0 \cdot (\vec{e}_x \cos(kz - \omega t) + \vec{e}_y \sin(kz - \omega t)) \tag{A1.2}$$

$$\vec{E}_R = E_0 \cdot (\vec{e}_x \cos(kz - \omega t) - \vec{e}_y \sin(kz - \omega t))$$

and that

$$\vec{E} = \vec{E}_L + \vec{E}_R = 2E_0 \cdot \vec{e}_y \sin(kz - \omega t) \tag{A1.3}$$

which represents a linearly polarized wave oscillating parallel to the y-axis.
Appendix 2 Light in anisotropic media.

2.1 Light propagating perpendicular to the optical axis in uniaxial crystals. Wave retarders.

Light incident on a quartz plate perpendicular to the optical axis is transmitted as 2 orthogonally polarized linear waves inside the plate, as shown in Figure A2-1.

![Image of wave fronts in a positive crystal](image)

Figure A2-1. The ordinary and extraordinary wave fronts in a positive crystal ($n_e - n_o > 0$). The difference between the 2 fronts is greatly exaggerated. The polarization state of the transmitted waves in some directions is also indicated. a) Optically active crystal  b) Not active

The 2 waves propagate with different velocities corresponding to different indices of refraction. Thus after passing through the plate with thickness $d$ the 2 waves have acquired a phase difference of:

$$\delta = \frac{2\pi}{\lambda_0} \cdot d \cdot \Delta n$$  \hspace{1cm} (A2.1)

Here $\lambda_0$ is the vacuum wavelength. A plate used in this configuration is called a wave retarder. If the incoming light is linearly polarized the 2 components will initially be in phase but shifted by $\delta$ at the exit. Thus, a retarder typically converts linear light to elliptical light. However, there are some important special cases:

- $\delta = 2m\pi$ Full wave plate. No noticeable effect on the light
- $\delta = (2m + 1)\pi$ Half wave plate. The outgoing light is still linearly polarized but the plane of oscillation is rotated.
- $\delta = (2m + 1)\pi/2$ Quarter wave plate. If the incoming light oscillates at 45° to the optical axis the outgoing light is circularly polarized

If a retarder plate is placed between 2 crossed polarizers we will in general retrieve some of the light, except for those wavelengths where the plate acts as a full wave plate. This will be clearly seen in the lab.

You will also use a quantitative measurement of the transmitted intensity as a function of wavelength to determine how $\delta$ and, from Eq. A2.1, $\Delta n$ depend on $\lambda$.  

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The set-up for this experiment is shown in Figure A2-2.

\[ \overrightarrow{E} = E_0 \cdot \vec{e}_{\parallel} \cdot \sin(\omega t) \]

This is, however, not convenient to handle the action of the retarder plate. Instead we describe the vertical light as a superposition of 2 waves parallel (\( e_{\parallel} \)) and perpendicular (\( e_{\perp} \)) to the optical axis, as shown in Figure A2-3.

\[ \overrightarrow{E} = E_0 \cos(\theta) \cdot \vec{e}_{\parallel} \cdot \sin(kz - \omega t) + \]
\[ E_0 \sin(\theta) \cdot \vec{e}_{\perp} \cdot \sin(kz - \omega t) \]

The effect of the retarder plate is to introduce a phase delay, \( \delta \), between these components. The observable wave at the exit of the plate will thus be:

\[ \overrightarrow{E} = E_0 \cos(\theta) \cdot \vec{e}_{\parallel} \cdot \sin(kz - \omega t + \delta) + E_0 \sin(\theta) \cdot \vec{e}_{\perp} \cdot \sin(kz - \omega t) \]

Only the projection of these fields on the transmission axis of the second polarizer will contribute to the final intensity. According to Figure A2-3 the field after the analyzer is:
\[ E = E_0 \cos(\theta) \cdot \cos(90 - \theta) \cdot \vec{e}_x \cdot \sin(kz - \omega t + \delta) - \]
\[ E_0 \sin(\theta) \cdot \cos(\theta) \cdot \vec{e}_x \cdot \sin(kz - \omega t) = \]
\[ \frac{1}{2} E_0 \sin(2\theta) \cdot \vec{e}_x \cdot \left( \sin(kz - \omega t + \delta) - \sin(kz - \omega t) \right) = \]
\[ E_0 \sin(2\theta) \cdot \vec{e}_x \cdot \cos\left(\frac{kz - \omega t + \delta}{2}\right) \cdot \sin\left(\frac{\delta}{2}\right) \]

This field gives a light intensity of:

\[ I \propto \left( E^2 \right) = E_0^2 \cdot \sin^2(2\theta) \cdot \sin^2\left(\frac{\delta}{2}\right) \cdot \left( \cos^2\left(kz - \omega t + \frac{\delta}{2}\right) \right) = \]
\[ \frac{1}{2} E_0^2 \cdot \sin^2(2\theta) \cdot \sin^2\left(\frac{\delta}{2}\right) \]

With \( I_o \propto \frac{1}{2} E_0^2 \) we obtain the final result:

\[ I = I_0 \cdot \sin^2(2\theta) \cdot \sin^2\left(\frac{\delta}{2}\right) \] \hspace{1cm} (A2.2)

Some consequences:

1. If \( \theta = 0 \) or \( 90^\circ \), i.e. if the incident light is parallel or perpendicular to the optical axis the final intensity is zero.
2. Maximum intensity is obtained for \( \theta = 45^\circ \).
3. If \( \delta = 2m\pi \), i.e. if the plate is a full wave plate the intensity is zero.
4. If \( \delta = (2m + 1)\pi \), i.e. if the plate is a half wave plate the intensity is maximized.

2.2 Light propagating parallel to the optical axis. Optical activity.

Linearly polarized light incident on an optically active media parallel to the optical axis is transmitted as a left- and right circularly polarized wave (Figure A2-1a). The 2 waves propagate with different velocities corresponding to different indices of refraction \( n_L \) and \( n_R \). Thus, after passing through the plate with thickness \( d \) the 2 waves have acquired a phase difference that is given by Eq. A2.1

The observable light after the plate is the superposition of the 2 circular waves, which is again a linearly polarized wave. But because of the acquired phase difference \( \delta \) the plane of vibration will be rotated an angle \( \beta \), as shown in Figure A2-4.

\[ \beta = \frac{\pi}{2} \cdot d \cdot \Delta n \] \hspace{1cm} (A2.3)

**Figure A2-4. Rotation of the plane of vibration in an optically active medium.**

If the plane of vibration is rotated to the right when looking towards the light the crystal is said to be dextero-rotatory (as in the figure above), whereas if the rotation is to the left it is called levo-rotatory.

If an optically active crystal is placed between crossed polarizers most of the light will be restored. However, you may block an arbitrary color by rotating the analyzer to an angle that is perpendicular to the new plane of oscillation. Figure A2-5 shows a set-up
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in which linearly polarized white light passes through an optically active medium and a second polarizer, perpendicular to the first, before it is sent into a spectrometer.

Figure A2-5. Set-up to investigate optical activity.

Let \( I_0(\lambda) \) be the wavelength dependent intensity after passing through the optically active medium, where each wavelength has undergone a rotation of \( \beta(\lambda) \). The angle of incidence on the transmission axis of the analyzer is therefore \( \pi/2 - \beta(\lambda) \). The intensity after the second polarizer according to Malus law is given by

\[
I(\lambda) = I_0(\lambda) \cos^2(\pi/2 - \beta(\lambda)) = I_0(\lambda) \sin^2(\beta(\lambda))
\]

Eq. A2.3 shows that the angle of rotation is directly proportional to the thickness, \( d \), of the crystal. A quantity that is independent of the thickness is the specific rotation or rotatory power \( \rho = \beta / d \). \( \rho \) exhibits a strong wavelength dependence both through the explicit appearance of \( \lambda \) in A2.3 and implicitly through the wavelength dependence of \( \Delta n \). At the yellow sodium light, \( \lambda = 589 \) nm, \( \rho \) for quartz is 21.7 \( ^\circ/\text{mm} \). A simple and useful approximation of the wavelength dependence of the specific rotation is:

\[
\rho = a + \frac{b}{\lambda^2} \quad (A2.4)
\]

Here \( a \) and \( b \) are experimentally determined constants

**Optical activity in solutions**

In a solid medium the anisotropic properties are typically due the regular pattern in which the constituent molecules are organized in space. However, some molecules exhibit anisotropic behavior also in a solution. A common example is a solution of ordinary sugar in water which is optically active and hence rotates the plane of linearly polarized light. For solutions the rotatory power, \( \rho \), is defined as the angle of rotation of a given wavelength after passing a 10 cm long cell containing a solution with a concentration of 1 g/ml. For sugar in water we have for example the values: \( \rho = 109.69^\circ \) at \( \lambda = 467.8 \) nm and \( \rho = 56.51^\circ \) at \( \lambda = 636.2 \) nm.
Appendix 3. Short manual to the program WinSpec

More information (in swedish) can be found at:
http://kurslab-atom.fysik.lth.se/Lars/sp300/Bruksanv/top.html

Choose grating and set the center wavelength on the CCD

Menu: Spectrograph/Move. Tab: Gratings
Choose between 300 or 2400 l/mm grating and give the desired wavelength.

View the spectrum by eye or measure using the CCD

Menu: Spectrograph/Move. Tab: Ports
Choose Front for the CCD or Side to view the spectrum through the exit port.
In addition you must select the operation mode of the shutter, which is placed in the entrance slit assembly.
Menu: Acquisition/Experimental Setup. Tab: Timing Box: Shutter Control.
Choose Normal for a CCD exposure or Disabled Opened to view the spectrum by eye.

Exposure time

Menu: Acquisition/Experimental Setup. Tab: Main

CCD-measurement

A measurement actually means 2 exposures; one background recording, with the shutter closed, and one with the shutter open for the selected exposure time. The program automatically subtracts the background. If you don’t change the exposure time you don’t need to record a new background.

Menu Acquisition/Acquire Background

Menu Acquisition/Acquire

Save the spectrum to a file

Menu File/Save As. Background subtracted data (channel number, intensity) are saved in binary form to a file with the extension .spe