Introduction

- Ionizing radiation
  - Ionizations
  - Excitations

- Direct Ionizing Radiation,
  - electrons, protons, alpha-particles, heavy nucleus, charged particles

- Indirect ionizing radiation
  - Photons and neutrons – uncharged particles that liberate direct ionizing particles in the interaction which in turn give rise to most of the remaining ionizations.

- Photons and neutrons
  - Low probability for interaction but when it occurs it results in a ‘drastic’ energy loss “statistical slowing-down”.

- Charged particles
  - Undergoes a large number of interactions each with a small energy deposition “continuous slowing-down”

- Photons indirectly ionizing
  - free secondary e− and large distances between the ionizations

- Heavy charged particles is densely ionizing
  - Large difference in biological effect and absorbed dose.
Interaction processes

- Processes within keV - MeV that is of importance for our applications.
  - Photo Absorption
  - Compton Scattering
  - Coherent Scattering
  - Pair Production
  - (Triplet Production)
  - (Photo Nucleus reaction)

Photoelectric effect

- High atomic number (Z), i.e. Lead
- Low photon energy

The Photoelectric Process

- The photon deliver the whole energy to an atomic e- (very low recoil energy)
  \[ h\nu = E_e - E_b \]
  \( h\nu \) = incoming energy of the photon
  \( E_e \) = kinetic energy of the e-
  \( E_b \) = binding energy of e-

- Results in a vacancy in the shell => characteristic X-ray emission and Auger electrons

- Process only with bounded electrons because nucleus receive an certain momentum.
Medical applications of Characteristic X-ray

- Lead in fingerbone can be measured by irradiating the finger by two 57Co (122 and 136 keV) sources mounted opposite to each other.

- The lead characteristic X-rays and incoherently scattered photons are detected using a high purity germanium detector.
Coherent scatter

- No energy loss for the photon
- Only change in direction

Compton scatter

- Energy loss for the photon
- Change in direction

The Compton Process

- The incoming photon interacts with a free electron or with an electron in an outer electron shell. The binding energy, $E_b$, is very small compared to $hv$.

- Relation between energy of the scattered photon and the incoming primary photon as function of scattering angle.
The Compton Process

Law of Energy: $h\nu = h\nu' + T$  \hspace{1cm} (1)

Law of momentum: $\frac{h\nu'}{c} = \frac{h\nu}{c} \cos \theta + p \cos \phi$  \hspace{1cm} (2)

$\frac{h\nu'}{\sin \theta} = p \sin \phi$  \hspace{1cm} (3)

Relativistic relations

\begin{align*}
\left\{ \left( mc^2 \right)^2 = \left( m_e c^2 \right)^2 + \left( pc \right)^2 \right\} & \Rightarrow \\
\Rightarrow \left( mc^2 + T \right) = \left( m_e c^2 \right) + \left( pc \right) & \Leftarrow \\
\Rightarrow \left( m_e c^2 + T \right) + 2m_e c^2 = \left( m_e c^2 \right) + \left( pc \right) & \Rightarrow \\
\Rightarrow \left( pc \right)^2 = T^2 + 2m_e c^2 T & \Rightarrow (4)
\end{align*}

The Compton Process

Eqn (2) och (3) results in

\begin{align*}
p^2 \cos^2 \phi &= \left( \frac{h\nu}{c} \right)^2 + \left( \frac{h\nu'}{c} \right)^2 \cos^2 \theta - \frac{2 \cdot h\nu \cdot h\nu'}{c^2} \cos \theta \\
p^2 \sin^2 \phi &= \left( \frac{h\nu'}{c} \right)^2 \sin^2 \theta
\end{align*}

Addition:

\begin{align*}
p^2 &= \left( \frac{h\nu}{c} \right)^2 + \left( \frac{h\nu'}{c} \right)^2 - \frac{2 \cdot h\nu \cdot h\nu'}{c^2} \cos \theta \\
\left( pc \right)^2 &= \left( h\nu \right)^2 + \left( h\nu' \right)^2 - 2 \cdot h\nu \cdot h\nu' \cos \theta & \Rightarrow (5)
\end{align*}

From eqn (1) $T = h\nu - h\nu'$ which in Eqn (4) results in

\begin{align*}
\left( pc \right)^2 &= (h\nu - h\nu')^2 + 2m_e c^2 \left( h\nu - h\nu' \right)
\end{align*}

The Compton Process

Put this into eqn (5) yield

\begin{align*}
(h\nu - h\nu')^2 + 2m_e c^2 (h\nu - h\nu') &= \left( h\nu \right)^2 + \left( h\nu' \right)^2 - 2h\nu \cdot h\nu' \cos \theta & \Leftarrow \\
\Rightarrow \left( h\nu \right)^2 + \left( h\nu' \right)^2 - 2 \cdot h\nu \cdot h\nu' + 2m_e c^2 (h\nu - h\nu') &= h\nu \cdot h\nu' - 2h\nu \cdot h\nu' \cos \theta & \Leftarrow \\
\Rightarrow m_e c^2 (h\nu - h\nu') &= h\nu \cdot h\nu' \left( 1 - \cos \theta \right)
\end{align*}

Division with $h\nu$ yield

\begin{align*}
\frac{m_e^2}{h\nu} (h\nu - h\nu') &= h\nu' \left( 1 - \cos \theta \right) \\
Define \hspace{1cm} \frac{h\nu}{mc} &= \alpha & \Rightarrow h\nu = h\nu' \cdot \alpha \left( 1 - \cos \theta \right) \\
h\nu' &= \frac{h\nu}{\alpha \left( 1 - \cos \theta \right)}
\end{align*}
Compton Scattering

- The photon energy before and after scattering
  \[ h' = \frac{h}{1 + \alpha (1 - \cos \theta)} \]
  \[ \alpha = \frac{h}{m_e c^2} \]

- The energy loss
  \[ h' - h = E = h' \alpha (1 - \cos \theta) \]

- The energy loss of the photon (Compton energy) is for a given
  \( h' \) determined by the scattering angle \( \theta \).

\[
\lambda' - \lambda = \frac{\hbar}{m_e c^2} (1 - \cos \theta)
\]

- Change in wave-length depend on \( \theta \).

- From the previous relations one see that for a given initial photon
  energy and knowing one of the following parameters

  \( E, h', \theta \) eller \( \phi \)

- Then, the others can be determined!
Pair production

- If $h\nu > 2m_e c^2 = 1.022$ MeV then it is energetically possible to create an $e^+e^-$ pair.
- The photon energy, $h\nu$, is transferred to rest masses for $e^+$ and $e^-$ and to kinetic energy.
- The process occurs close to a charged particle (nucleus) which takes up a certain momentum.
- Threshold energy is $2m_e c^2$ MeV for interaction with a nucleus.
- Threshold energy is $4m_e c^2$ MeV for interaction with an $e^-$ (triplet of $e^- + e^+ +$ recoiling $e^-$)

\[ h\nu = 2m_e c^2 + E_{e^-} + E_{e^+} + E_{\text{rec}} \]

Photon attenuation

- $\Phi_0$ photon fluence,
- $\mu$ attenuation coefficient,
- $d$ thickness
**Photon attenuation**

\[ dN = -\mu \cdot N(x) \cdot dx \]

\[ \frac{dN}{dx} = -\mu \cdot N(x) \]

\[ N(x) = N_0 e^{-\mu x} \]

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**Photon attenuation**

\[ \frac{N(x)}{N_0} = e^{-\mu x} \] Probability for transmission > 0

Mean-Free Path = mfp = \( 1/\mu \)

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**Attenuation coefficient**

- **Type of processes**
  - Photo-electric effect
  - Incoherent Scattering
  - Coherent Scattering
  - Pair-production
  - Triplet-production
  - Nuclear photoeffect

- **Total mass-absorption coefficient is the sum of all interaction processes**

\[ \mu = \mu_{\text{photo}} + \sigma_{\text{Compton}} + \sigma_{\text{coherent}} + \sigma_{\text{pair}} + \sigma_{\text{triplet}} + \sigma_{\text{nuclear}} \]
Data on the internet!

http://www.nist.gov/pml/data/xcom/index.cfm

Cross-section as function of energy

Photoelectric effect

* High atomic number (Z), i.e. Lead
* Low photon energy
The Photoelectric Process

- Highest likelihood for tight bounded electrons (hv > EB).
- 80% of all interactions by K-electrons.
- Defined for a scatter angle θ and a solid angle dΩ.

\[ \sigma_{\gamma,e} \]

\[ \sigma_{\gamma,e} = 4 \cdot 10^{-27} \cdot s(Z, \nu) \cdot (Z - 0.3)^{5} \cdot (\nu)^{-7/2} \] m²

Probability for photo process increase rapidly with increasing Z and decreasing energy.

The atomic photo-effect cross section

Coherent scatter

- Low atomic number (tissue, water)
Thompson cross-section

- Starting point for the classical description of coherent x-ray scattering is the Lorentz equations
  - Relates the force on an unbound electron to the amplitude of an electromagnetic wave with which it interacts.
  - The electromagnetic wave train causes the electron to oscillate.
  - The electron thus giving rise to its own radiation, with a frequency identical to that of the incoming wave:
    \[ \frac{d\sigma}{d\Omega} \text{(Thomson)} = \frac{e^4}{2\hbar^2} (1 - \cos^2 \theta) \]

- Such considerations lead to the Thomson form of the differential (with angle) scatter cross section of a free electron

\[ \sigma_{\text{Thomson}} = \frac{e^4}{2\hbar^2} (1 - \cos^2 \theta) \]

\( r_0 \) is the classical electron radius, \( \theta \) is the angle between the incident photon momentum and the scattered photon momentum.

Coherent Cross-section

- Thomson cross-section per electron
  \[ \frac{d\sigma}{d\Omega} \text{(electron)} = \frac{e^4}{2\hbar^2} (1 + \cos^2 \theta) \]

- Cross-section per atom
  - Independent of energy – not realistic
  \[ \frac{d\sigma}{d\Omega} \text{(atom)} = \frac{e^4}{2\hbar^2} (1 + \cos^2 \theta) \cdot Z \]

- Formfactor include energy by the parameter \( x \)
  \[ \sigma(x) = \frac{e^4}{2\hbar^2} (1 + \cos^2 \theta) F^2(x, Z) \]

- Total cross-section
  \[ \sigma = \int \frac{1}{2} \frac{d\sigma}{d\Omega} (\theta, Z) \, d\theta d\phi \]

Coherent Scattering

- Diagram showing the coherent scattering for different materials such as Carbon.
Compton scatter

\[ h' = \frac{h \nu}{1 + \alpha (1 - \cos \theta)} \]
\[ \tan \varphi = \frac{\sin \theta}{(1 + \alpha)(1 - \cos \theta)} \]

The Klein-Nishina Cross-section

- Quantum mechanical extension to the Thomson Cross-section
- As energy decreases \( \alpha (=hv/mc^2) \) tends towards zero and regain Thompson differential cross-section

\[ \frac{d\sigma}{d\Omega} = \frac{d\sigma_{th}}{d\Omega} \frac{\alpha^2}{2} (1 + \cos \theta) F_{kn} \]

where

\[ F_{kn} = \left( \frac{1}{1 + \alpha (1 - \cos \theta)} \right) \left( 1 + \frac{\alpha^2 (1 - \cos \theta)^2}{1 + \alpha (1 - \cos \theta)(1 + \cos \theta)} \right) \]
The Klein-Nishina Cross-section

- Differential cross-section per electron for a Compton process where the photon is scattered an angle \( \theta \) in the solid angle element \( d\Omega \) is given by:

\[
d\sigma_{e,e}^c = \frac{r_e^2}{2} \left( \frac{h\nu}{h\nu'} \right)^2 \left( \frac{h\nu'}{h\nu} + \frac{h\nu - h\nu'}{\sin^2 \theta} \right) d\Omega
\]

\[
\sigma_{e,e}^c = Z \cdot \sigma_{\gamma,e}^c
\]

Direction distribution for photons

- Distribution for low energies are symmetrical around 90\(^\circ\)
- Increase in the forward direction for higher energies

Direction distribution for photo electrons

- Low energies means electron emissions symmetrical around 90\(^\circ\).
- Photons incoming with higher energies results in photo electrons that are forward directed.
Cross-section per atom for Klein-Nishina

- Cross-section for a Compton process is equal for all electrons and independent of Z (assuming that the binding energy are neglectable)

\[ \sigma^a_{\gamma,\gamma e} = Z \cdot \sigma^e_{\gamma,\gamma e} \]

- Additional corrections necessary when the photon energy is in the order of the binding energies of the atomic electron.

Pair production

- High energies (hv>1.022 MeV)
- High atomic number

Bethe cross-section for Pair Production

- Valid in the range \( hv < 15 \text{ MeV} \).

\[ \sigma^e_{\gamma,\gamma e} = \frac{1}{137} \left( \frac{e^2}{4\pi\alpha m_e c^2} \right) Z^2 \left( \frac{28}{9} \ln(2\alpha) - \frac{218}{27} \right) \]

- For higher energies the process can occur at larger distances from the nucleus
- Increase slowly with photon energy
Bethe cross-section for pair production

For very high energies:

\[
\sigma_{\gamma,e^+e^-} = \frac{1}{137} \left( \frac{e^2}{4\pi\epsilon_0 m_e c^2} \right) Z^2 \left( \frac{28}{9} \ln(183 - 2^{1/3}) - \frac{218}{27} \right)
\]

No energy dependence - no \( \alpha \)

\[ \sigma_{\gamma,e^+e^-} \sim Z^2 \]

Same Z dependence for all energies

Energy dependence

\( \mu - (\hbar \nu)^{7/2} \) If photo absorption dominates!

\( \mu \)

Decrease slowly with increasing energy of Compton scattering dominates.

\( \mu \)

Increase slowly with \( \hbar \nu \) if pair production dominates.

Z dependence

\( \mu - Z^5 \) If photo absorption dominates

\( \mu - Z \) Independent of \( Z \) if Compton scattering dominates

\( \mu - Z^2 \) If pair production dominates
Dominating cross section

Differential cross-sections - Hydrogen
- Low-Z material

Differential cross-sections – Iodine
- Z=52
- K-Xrays at 30 keV
Differential cross-sections - Lead

- $Z=82$
- K-Xrays at 80 keV
- L, M edges visible

Neutrons

- Neutrons interacts by
  - Elastic scattering by direct collision with a nucleus
  - Inelastic scattering which means that the neutron is absorbed by the nucleus that will become in an excited state and thereby emit a neutron in a different direction.
  - Absorption where the nucleus is excited. The excess of energy is emitted as gamma and/or particles
- For each type above a cross-section can be defined. The total is the sum of
- Compare with photons

Cross-sections