Roadmap transitions

- Einstein coefficients $A_{21}, B_{12}$ and $B_{21}$ (H2, #31)
  Foot Ch 1.7, SP 7.4, 7.5
- Basic conditions for laser action
- Much more about lasers (not covered here):
  Spectrophysics Ch. 14
  Wikipedia,
  Atomic Physics Home page (Popular description in Swedish)
  http://www.atomic.physics.lu.se/research/,
  Lecture by Prof Anne L'Huillier (Monday 11/2)
  Many elective courses given by the division

- Lifetime and Intensity

- Selection rules
  Foot Table 5.1, SP 2.4.4

- Relative intensities in $LS$ multiplets
  (H2, #33) and two-electron lab
Boltzmann population distribution at thermal equilibrium

\[ \frac{N_2}{N_1} = \frac{g_2}{g_1} \cdot e^{-\Delta E/kT} \]

\( g = \) statistical weight
\( g = 2J + 1 \) or \( 2F + 1 \)

\( T_1 \)
\( T_2 > T_1 \)
Degeneracy – statistical weight, \( g \), in a pd-configuration.

\[
\begin{align*}
g(E) &= 60, \\
g(\ ^3D) &= (2L + 1) \cdot (2S + 1) = 15 \\
g(\ ^3D_3) &= 2J + 1 = 7
\end{align*}
\]
Planck’s Radiation Law

Assume that light can only be absorbed or emitted in discrete quantities (photons) where the energy depends on frequency as:

\[ E = h \cdot f = h \cdot \frac{c}{\lambda}, \quad h = 6,62 \cdot 10^{-34} \text{ Js} \]

The energy density per frequency interval, \( \rho \), is then given by:

\[
\rho(f) = \frac{8\pi hf^3}{c^3} \cdot \frac{1}{e^{hf/kT} - 1},
\]

\[ [\rho] = 1 \text{ J/(m}^3\times\text{Hz)} = 1\text{Js/m}^3 \]
Einstein coefficients

\[ \rho(f) \]

\[ A_{21}N_2 + \rho B_{21}N_2 = \rho B_{12}N_1 \Rightarrow \quad \begin{cases} g_2 B_{21} = g_1 B_{12} \\ g_2 A_{21} = \frac{8\pi hf^3}{c^3} g_1 B_{12} \end{cases} \quad \text{(Exercise 32)} \]

If \( g_1 = g_2 \) then

\[ \begin{cases} B_{21} = B_{12} \\ A_{21} \sim f^3 \cdot B_{21}, \sim f^3 \cdot B_{12} \end{cases} \]

Consequences:

- Drive hard and saturate a transitions, i.e. \( N_1 = N_2 \)
- Laser actions requires \( N_2 > N_1 \) i.e. an inverted population
- Difficult to obtain laser action at short wavelengths due to the \( f^3 \) scaling
Selection rules and metastable levels in He

Fig. 2.9. The energy level structure of He.

Observations of so-called spin forbidden transitions, i.e. where $\Delta S \neq 0$, is one sign of intermediate coupling effects. The $1s^2 \, ^1S_0 - 1s2p \, ^3P_1$ has indeed been observed in all He-like systems.
Laser problem 1: Inverted population

Always some ”trick”.
For example optical pumping or the HeNe-scheme. The latter uses a near coincidence in energy between $2s^1,3S$ in He and $4s$ and $5s$ in Ne which opens a selective collisional excitation of the latter levels in Ne, thereby obtaining an inverted population relative to lower lying levels.

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**Diagram**: 
- **Helium**
  - Ground State: $1^1S$
  - Metastable: $2^3S$
  - Collision:
    - $2^1S$ to $2^3S$

- **Neon**
  - Ground State: $1S^22S^22P^6$
  - $3S$ to $3P$
  - $5S$ to $4S$ with $100\,\text{ns}$
  - $4S$ to $1152\,\text{nm}$
  - $3S$ to $3391\,\text{nm}$
  - $3391\,\text{nm}$ to $632.8\,\text{nm}$ Laser light
  - Wall collisions:
    - $1S^22S^22P^6$ to $1^1S$
  - Rapid deexcitation channel: $10\,\text{ns}$
Laser problem 2: High intensity
(Fabry-Perot lecture in lab preparation)

\[ \Delta \sigma_{fsr} = \frac{1}{2 \cdot L \cdot n}, \]

\[ (\Delta \delta)_{\text{min}} = 2 \cdot \frac{1 - R}{\sqrt{R}}. \]
Spectral line intensity.

\[ n_{ij} = A_{ij} \cdot N_i \]

Where \( n_{ij} \) is the number of emitted photons per second in \( 4\pi \) steradian (full sphere).

The number of \textbf{detected} photons, the \textit{intensity}, is then

\[ I_{ij} = \varepsilon(\lambda_{ij}) \cdot A_{ij} \cdot N_i \]

where \( \varepsilon(\lambda) \) takes care of the solid angle subtended by the detector and all, wavelength dependent, detector efficiencies. Here the intensity is measured in \textbf{number of photons per second and unit area}, meaning that the normal unit \( \text{W/m}^2 \) is obtained by multiplying by the energy \( hf \) of one photon.
Almost all information we have about our surroundings comes from the analysis of light.
Chi Lupi is a blue giant star in the constellation Lupus, 195 light years away.

Isotope anomaly of Hg in the atmosphere of χ-Lupi.
Absorption - classical.

The incoming light (electric field) induces an oscillating electric dipole moment $\overline{d} = -q \cdot \overline{E}$

Damped and driven harmonic oscillator

$$\frac{d^2 r}{dt^2} + \frac{b}{m} \cdot \frac{dr}{dt} + \omega_0^2 \cdot r = \frac{qE_0}{m} \cdot \cos \omega t$$

$$r(t) = A \cdot \cos(\omega t - \delta)$$

$$A = \frac{qE_0}{\sqrt{m^2 (\omega_0^2 - \omega^2)^2 + b^2 \omega^2}}$$
Selection rules E1 (electric dipole) transitions

\[ \Delta J = 0, \pm 1 \] not 0 to 0

Exercise 17

Only one electron can change orbital, i.e. \( n\ell \)

Highly unlikely that two electrons would rearrange themselves simultaneously

\( \hat{r} = \) one-electron operator

\[ \Delta \ell = \pm 1 \]

\( \hat{r} \) has odd parity and \( Y_{\ell,m}(\theta,\phi) \) has \((-1)^{\ell}\)

If perfect LS-coupling, i.e. real states = basis states

\[ \Delta S = 0 \]

\( \hat{r} \) does not include spin, thus can't change it

\[ \Delta L = 0, \pm 1 \] ej 0 till 0

Follows from \( \Delta J \) and \( \Delta S \)
Li-sequence
\[2s^2 S_{1/2} - 2p^2 P_{1/2,3/2}\]
Relative intensities in the Be-sequence

$2s3s \ ^3S_1 - 2s3p \ ^3P_{0,1,2}$ transitions
Relative intensities in some LS multiplets

The (normally) very intense “diagonal” in the multiplets is shown in red

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<th>$^2P_{1/2}$</th>
<th>$^2P_{3/2}$</th>
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<th>$^2F_{5/2}$</th>
<th>$^2F_{7/2}$</th>
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<td>1</td>
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Relative intensities in a $^3\text{D} - ^3\text{F}$ multiplet and the LS sum rules

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<th>$^3\text{D}_2$</th>
<th>$^3\text{D}_3$</th>
<th>$\Sigma$ int.</th>
<th>$g_L = 2J+1$</th>
<th>$(\Sigma$ int.)/$g_L$</th>
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<td>35</td>
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<td>45</td>
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<td>$^3\text{F}_3$</td>
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<td>35</td>
<td></td>
<td>315</td>
<td>7</td>
<td>45</td>
</tr>
<tr>
<td>$^3\text{F}_4$</td>
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<td></td>
<td></td>
<td>405</td>
<td>9</td>
<td>45</td>
</tr>
<tr>
<td>$\Sigma$ int.</td>
<td>189</td>
<td>315</td>
<td>441</td>
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<tr>
<td>$g_U = 2J+1$</td>
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<td>5</td>
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<tr>
<td>$(\Sigma$ int.)/$g_U$</td>
<td>63</td>
<td>63</td>
<td>63</td>
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</tr>
</tbody>
</table>

The sum of all intensity TO a given LOWER level is proportional the statistical weight ($2J+1$) of the level. The constant is the same for all levels of the lower LS term.

The sum of all intensity FROM a given UPPER level is proportional the statistical weight of the level. The constant is the same for all levels of the upper LS term.