Solutions Atomic Physics 1

1a. The *s*- and *p*-electrons penetrate the core more than *d*-electrons and thus become more tightly bound

1b. $T(4\mathrm{p}\ ^2\mathrm{P}_{3/2}) = \dfrac{R\cdot 1^2}{(4-\delta_{4\mathrm{p}})^2} = E_{\mathrm{ion}} - E_{\mathrm{ex}}(4\mathrm{p}\ ^2\mathrm{P}_{3/2}) \Rightarrow \delta_{4\mathrm{p}} = 1.765$ with $R = R_\infty$.

$\delta_{5\mathrm{p}} \approx \delta_{4\mathrm{p}} \Rightarrow E_{\mathrm{ex}}(5\mathrm{p}\ ^2\mathrm{P}_{3/2}) = E_{\mathrm{ion}} - \dfrac{R\cdot 1^2}{(5-\delta_{4\mathrm{p}})^2} = 24524\ \mathrm{cm}^{-1}$ (exp: 24720 cm^{-1})

1c. $\Delta E_{\mathrm{fine}} \square (n^*)^{-3} = (n-\delta)^{-3} \Rightarrow \dfrac{\Delta E(5\mathrm{p})}{\Delta E(4\mathrm{p})} = (\dfrac{4-\delta}{5-\delta})^3 \Rightarrow \Delta E(5\mathrm{p}) = 19\ \mathrm{cm}^{-1}$

(exp 18.76 cm^{-1})

2a. $\dfrac{1}{\lambda_{\mathrm{H}\beta}} = R_{\mathrm{H}}\cdot(\dfrac{1}{2^2} - \dfrac{1}{4^2}) \Rightarrow \lambda_{\mathrm{H}\beta} = 4862.76$ Å, $R_{\mathrm{H}} = 109677\ \mathrm{cm}^{-1}$.

2b. $f = c/\lambda \Rightarrow \Delta f_{\mathrm{D}} = \dfrac{c}{\lambda^2}\Delta\lambda_{\mathrm{D}} = 4.440\cdot 10^{10}$ Hz.

$\Delta f_{\mathrm{D}} = \mathrm{const}\cdot\sqrt{\dfrac{T}{M}} \Rightarrow T = 10000$ K.

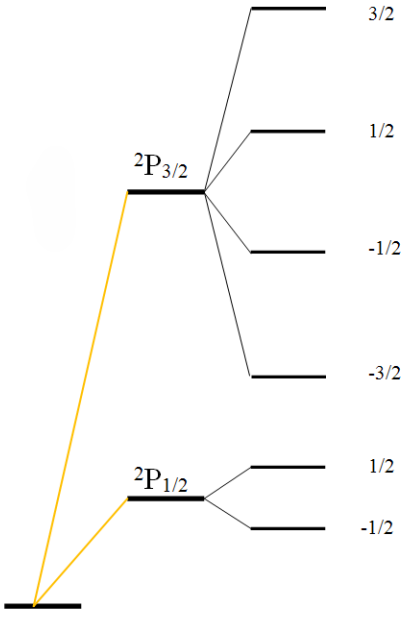
3. The magnetic energy level separation in the 3p term is shown schematically in the figure together with the yellow resonance lines. The g_{J} - factor is 4/3 in $^2\mathrm{P}_{3/2}$ but only 2/3 in $^2\mathrm{P}_{1/2}$.

$E_{3/2} - \mu_{\mathrm{B}}\cdot B\cdot\dfrac{4}{3}\cdot\dfrac{3}{2} = E_{1/2} + \mu_{\mathrm{B}}\cdot B\cdot\dfrac{2}{3}\cdot\dfrac{1}{2} \Rightarrow$

$\Delta E(3/2 - 1/2) = \mu_{\mathrm{B}}\cdot B\cdot(2+\dfrac{1}{3}) \Rightarrow$

$B = 17$ T.

Clearly the Zeeman model breaks down long before such high fields are reached. Actually the perturbation treatment assumes that the magnetic energy is very small compared to the fine structure energy. So this is only a numerical value giving some quide to what is a weak field, i.e. B << 17 T!

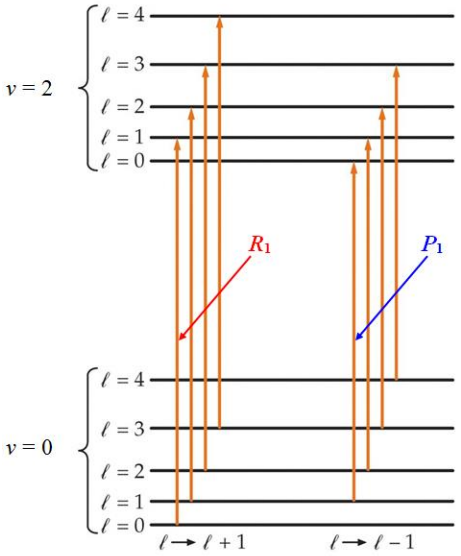


4. The R_1 and P_1 transitions are shown in the figure. The rotational energy in a diatomic molecule is given by:

$$E_{rot} = \frac{\hbar^2}{2\mu r_0^2}\ell(\ell+1) \equiv B\ell(\ell+1),$$

where r_0 is the equilibrium distance between the atoms and μ is the reduced mass. In CO

$$\frac{1}{\mu} = \frac{1}{12} + \frac{1}{16} \Rightarrow \mu = 6.86 \text{ u} = 1{,}138 \cdot 10^{-26} \text{ kg}.$$

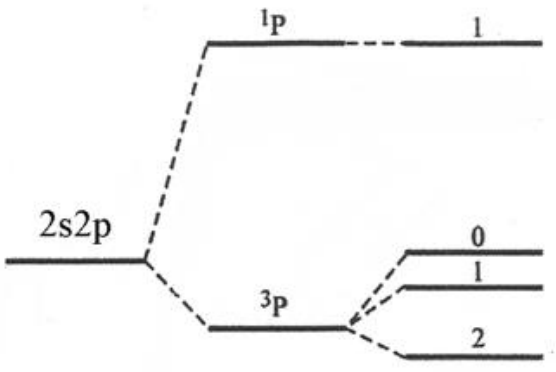


Hence the wavenumbers for the R_1 and P_1 transitions are given by:

$\sigma_{R_1} = \sigma_{00} + 2B$ and $\sigma_{P_1} = \sigma_{00} - 2B$ if we assume that the rotational constants are (approximately) equal in the 2 vibrational states. From this follows that:

$$B = \frac{\sigma_{R_1} - \sigma_{P_1}}{4} = 1.898 \text{ cm}^{-1} = 0,2352 \text{ meV} = 3{,}769 \cdot 10^{-23} \text{ J and } r_0 = \frac{\hbar}{\sqrt{2\mu B}} = 1{,}13 \text{ Å}$$

5. The energy structure in a $1s^2 2s2p$ configuration under LS-coupling is shown schematically.



The different energy contributions are briefly as follows:

Configuration: Large electrostatic interactions in the central field model.
Terms: Non-central parts of the electrostatic repulsion between the electrons.
Levels: Magnetic spin-orbit interaction.

6a. See any quantum mechanics book.

6b. Operators that commute can have well-defined values simultaneously and have common eigenfunctions