THE ZEEMAN EFFECT 190529

Introduction

Quantum mechanics tells us that angular momenta, $\bar{J}$, are quantized both regarding magnitude, $J^2 = \hbar^2 J(J+1)$, and direction, $J_z = M_J \cdot \hbar$, as illustrated in Figure 1. The directional quantization is normally not noticeable in the energy level structure of an atom since the "z"-direction is an arbitrary direction (isotropic space), and all states that differ only in the value of M_J have the same energy. The level is said to be 2*J* + 1 fold degenerate. However, if the atom is placed in an external magnetic field the degeneracy is broken due to energy differences between states that are aligned with the external field and states that are not. Thus, the different M_J - states obtain a small additional energy:

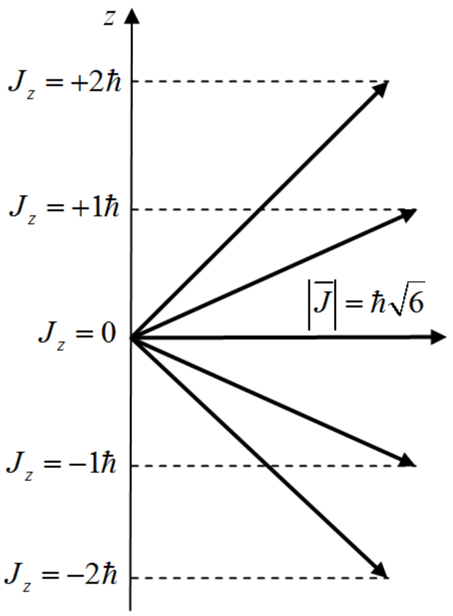


Figure 1. Vector model picture of a quantized angular momentum with $J = 2$.

$$E_{\text{mag}} = g_J \mu_B B M_J \,.$$

Here g_J is the Landé factor for the level, μ_B is the Bohr magneton, *B* is the magnetic field strength and M_J is the secondary total angular momentum quantum number. Furthermore, light emitted from an excited atom becomes polarized with an oscillation mode that depends on how the M_J quantum numbers change as well as the direction of observation relative to the external magnetic field. Viewed perpendicular to the *B*-field light is linearly polarized; oscillating parallel to the field for $\Delta M_J = 0$ transitions and perpendicular for $\Delta M_J = \pm 1$. Viewed along the field direction $\Delta M_J = \pm 1$ transitions give rise to circularly polarized light.

Preparation

Study carefully the theory of the Zeeman effect in e.g. Thorne *et al*, Spectrophysics[1], Ch. 3.9.1 or Foot, Atomic Physics[2], Ch. 1.8 and 5.5. Since the Zeeman splitting of spectral lines is small, we need an instrument with very high spectral resolution to measure it. One such instrument is the Fabry-Perot interferometer described in Spectrophysics 13.1 - 13.4, Pedrotti *et al*, Introduction to Optics[3] and also briefly in Appendix 1, below. You must study in detail (at least) the last section of Appendix 1, where the evaluation of the experiment is described. Recapitulate the description, production and manipulation of polarized light. This is very briefly outlined in Appendix 2. If you are not familiar with the concept of polarization, read the section "Linear Polarization" on p. 1115 of University Physics with Modern Physics and "Circular and Elliptical Polarization" on p. 1121 in the same book (e-book available through the physics library at https://lubcat.lub.lu.se/).

1 A. Thorn, U. Litzén and Se. Johansson, Spectrophysics, Springer

2 C. Foot, Atomic Physics, Oxford master series in Physics

3 F.L. Pedrotti, L.M. Pedrotti and L.S. Pedrotti, Introduction to Optics, Pearson

4 H.D. Young, R.A. Freedman, A.L. Ford, University Physics with Modern Physics, Global ed., Pearson

Preparatory exercises (Serious attempts must be made on all exercises)

1. The ground configuration in neutral Cd is $5s^2$ and the first excited configuration is 5s5p. In the experiment you will, among other lines, see the transition 5s5p ^3P - 5s6s ^3S.

   a) Give the *LS* notation for the possible transitions between these terms.

   b) You will find a green (λ = 508.582 nm), a turquoise (λ = 479.992 nm) and a blue (λ = 467.816 nm) line. Which of the transitions above correspond to the different colors?

2. **This exercise is essential to the lab and a solution must be presented before you are allowed to continue.**

   In the experiment you will study the transitions 5s5p $^1P_1$ - 5s5d $^1D_2$, 5s5p $^3P_2$ - 5s6s $^3S_1$, 5s5p $^3P_1$ - 5s6s $^3S_1$ and 5s5p $^3P_0$ - 5s6s $^3S_1$ in a weak magnetic field.

   a) Derive the Landé *g* factor assuming *LS* coupling for the levels involved in the four transitions.

   b) Draw large and nice diagrams showing the different Zeeman components that each of the 4 lines (not levels) split into in the magnetic field in the manner of Figure 3.16 in Spectrophysics or Figure 5.13 in Atomic Physics. Thus, choose a relative energy scale, with zero at the energy of the transition without magnetic field, and show the splittings in units of $\mu_B B$ along the x – axis. Let all Zeeman components have the same intensity.

   c) What is the state of polarization of each of the components? (Hint: ΔM is positive if M_J of the upper state > M_J of the lower state)

   d) Which components do you expect to see in a direction parallel to the magnetic field?

3. Let B = 0.5 T. How large is the smallest splitting between the components derived above?

   a) Expressed in eV

   b) Expressed in cm^{-1}

   c) Expressed in nm

4. Use Appendix 1 to answer the following. A Fabry-Perot interferometer operating in air have mirror surfaces with a reflectance of R = 0.85 and separated by 3.085 mm. We use a light source with a wavelength of 500 nm.

   a) What is the free spectral range expressed in cm^{-1} and in nm.

   b) What is the line width expressed in cm^{-1} and nm.

   c) Does the size of the rings increase or decrease in higher spectral orders?

5. Use Appendix 2 to answer the following. What is the polarization of light when the electric field is described by the expressions below?

   a) $\overline{E} = E_0 \cdot (\overline{e}_x \cdot \sin(kz - \omega t) + \overline{e}_y \cdot \cos(kz - \omega t))$

   b) $\overline{E} = 5 \cdot \overline{e}_x \cdot \sin(kz - \omega t + \pi/2) + 3 \cdot \overline{e}_y \cdot \sin(kz - \omega t)$

Experiments

Setup

Neutral cadmium atoms have the ground configuration $5s^2$ and the system of excited levels $5sn\ell$. In the experiment you are going to study the 4 lines

$5s5p\ ^1P_1 - 5s5d\ ^1D_2$ 643.8 nm (red)

$5s5p\ ^3P_2 - 5s6s\ ^3S_1$ 508.6 nm (green)

$5s5p\ ^3P_1 - 5s6s\ ^3S_1$ 480.0 nm (turquoise)

$5s5p\ ^3P_0 - 5s6s\ ^3S_1$ 467.8 nm (blue)

The cadmium light is emitted from a spectral lamp placed between the poles of an electromagnet, where the field is directly proportional to the current. Light then passes through a Fabry-Perot interferometer and the interference pattern is studied either by eye or with a CCD camera. The setup is shown in Figure 2. The lenses have focal lengths of 50, 300 and 50 mm as indicated in the figure.

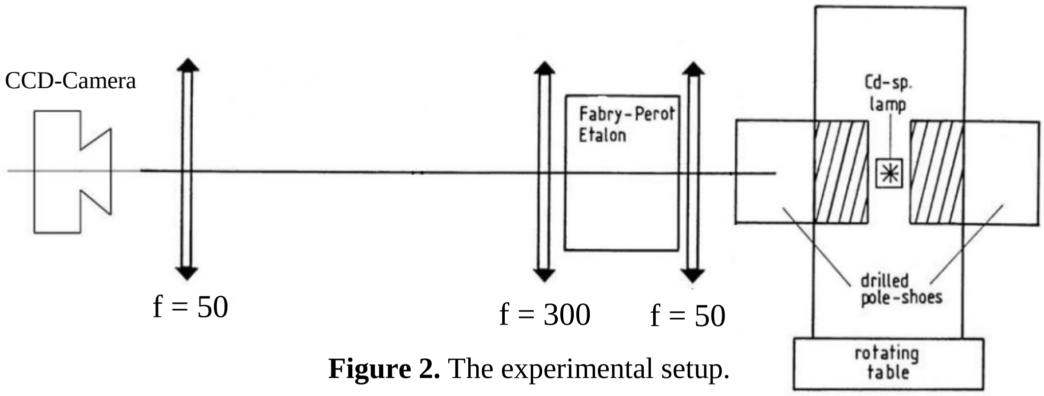


Figure 2. The experimental setup.

To isolate a certain wavelength from the Cd-light (to look at one transition at the time) we need external narrow band interference filters. Figure 3 shows the transmission curves for the filters for the blue and the turquoise light.

The polarization of the Zeeman components of light emitted perpendicularly and parallel to the magnetic field is investigated using a polarizer and an achromatic λ/4 plate (see Appendix 2).

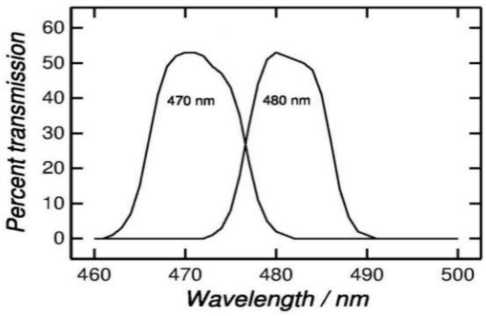


Figure 3. Transmission curves for the interference filters used. Data from Thorlabs, http://www.thorlabs.com/.

Qualitative studies (the most important part!)

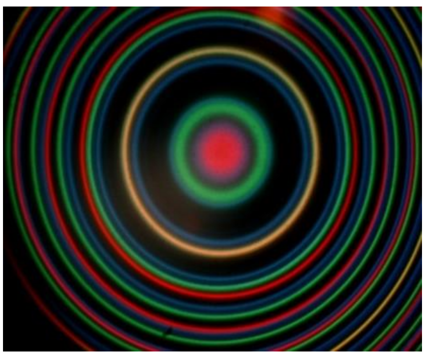


Figure 4. Fabry-Perot pattern with no filters or magnetic field.

Start by viewing the Fabry-Perot interference patterns directly by putting your eye immediately behind the last lens. Optimize the positions of the lenses to see the pattern as clearly as possible. What happens when you use different filters or different magnetic field strengths? Study the transitions from both the transverse and longitudinal direction with regards to the magnetic field. Do your observations agree with the predictions in preparatory exercise 2? Is something missing in one direction and in that case why? Then, insert the polarizer and other necessary instruments and verify the polarization of the σ-components you determined in exercise 2.

Quantitative studies

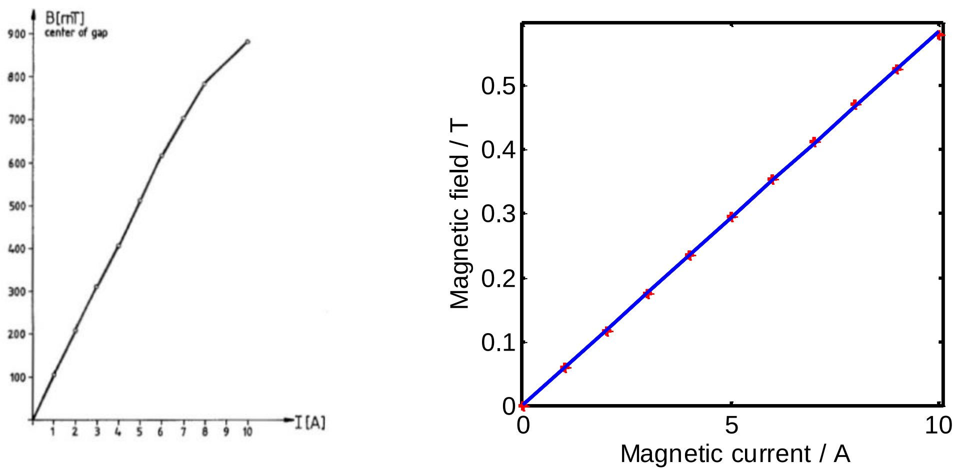
Carefully adjust and center the CCD camera on the pattern. The software *uEye Cockpit* allows you to capture images with the camera. The brightness can be adjusted by tuning certain camera parameters (for example the exposure time) that you can find through: *tool → camera*.

Perform a set of measurements in the direction that gives you the clearest image of the σ - splittings, for each transition. If you use the transverse direction, you need to use one additional optical instrument. Take data for several magnetic currents between 0 and, at most, 4A (NOTE! For one of the setups the maximum is 3A) and capture the ring pattern for each setting to a file. You will need several points in order to perform the fitting, so do not take too few values for the magnetic current!

Figure 5. Fitting of the Fabry-Perot rings with the 3-point-circle tool in *Motic Images Plus*.

As a preparation for the quantitative analysis according to eq. (7) in Appendix 1, use the software *Motic Images Plus*, load an image, then go to the "Measurements" menu and choose the "3-point circle" to determine the diameter of the observed rings for each magnetic current, as illustrated in Figure 5. Make sure to save these images to a USB-stick for the analysis and the report writing!

Use eq. (7) in Appendix 1 and your measured diameters and plot, in the same diagram, the Zeeman splitting, $\Delta\sigma$, as a function of the current through the magnet for the different transitions. The magnetic field is directly proportional to the current, as shown in Figure 6 for currents up to 5 A. Verify that the Zeeman splitting is, indeed, directly proportional to the magnetic field. To compare the numerical value of the splittings to the theoretical prediction you need the approximate proportionality constants which are 0.1 T/A for the Phywe magnets and 0.058 T/A for the "large magnet".



a b

Figure 6. Magnetic field as a function of current for the a) Phywe magnets and b) "Large magnet".

To eliminate also the need to accurately know the thickness of the spacer between the plates in eq. (7), show that the ratio of the slopes of the red and blue lines is accurately predicted by the quantum numbers and g_J - factors involved, see preparatory exercise 2.

Fitting your results

Since you expect a linear relation between magnetic current and splitting, it is recommended to use a linear fit to the data. It is allowed to use built-in functions in for example MATLAB for this, but you can also use a manual least-squares method by trying out different values on k and m for $y = kx + m$ and see which values minimize $\sum_i^N (y(x_i)_{measured} - y(x_i)_{fit})^2$, where N is the total number of data points taken. From this you can also find the standard deviation S:

$$S = \sqrt{\frac{\sum_i (y(x_i)_{measured} - y(x_i)_{fit})^2}{N}}$$

The value of S might be interesting to compare to your own estimation of the size of the uncertainty in your method.

Checklists

Before you leave the lab

Before you leave the lab, make sure that you have done all of the following required steps:

- Made sure you understand the function and purpose of all instruments used.
- By eye observed the effect of the longitudinal vs transverse direction of the optical bench with regards to the magnetic field and the effect of changes in the magnetic field strength.
- Confirmed the polarization of the σ-lines and fully understand the procedure.
- Taken enough data points (i.e. enough current values have been used) to measure the Zeeman splitting of the σ-lines of <u>all</u> transitions, unless otherwise instructed by the supervisor, and properly fit a function to your data.
- Measured the diameters D_1 and D_2 for the most appropriate image as well as all necessary D_a and D_b, and saved these values (preferably also all images) to a USB stick.

Before handing in your report

When writing a report, there are some details that are often overlooked, so take a look through the short checklist below to ensure that you have not forgotten any of these points:

- Have you answered all the questions asked in the lab manual? (these questions are there to guide your thinking and start your analytic train of thought!)
- Are your plots easy to understand and do you guide the reader through them and point out important features etc?
- Do all of your figures have figure captions that are independent enough from the text that you would be able to interpret the figure without the text?
- Do all of your facts regarding the background/theory have a correct reference to a trustworthy source?

Appendix 1:

The Fabry-Perot interferometer, free spectral range, line width and data reduction for the Zeeman lab.

You have most likely discussed the phenomenon of interference in thin films, shown in Figure A1-1, in some earlier course. In many cases the reflectance of the surfaces is low and only the first two rays have a substantial impact on the output. However, if the reflectance is high we must sum the contribution from (very) many rays, a phenomenon called multiple beam interference. This is the basis for the Fabry-Perot interferometer. A typical interferometer is shown schematically in Figure A1-3. Two plane glass plates with highly reflecting surfaces facing each other are separated by a distance *d*. Light from an extended source (see Figure A1-2) pass through the system, as in Figure A1-3. The light is reflected many times between the plates and at each reflection a small part is transmitted. Each transmitted wave acquires a phase difference of δ relative to the previous one due to the extra "round trip" between the plates:

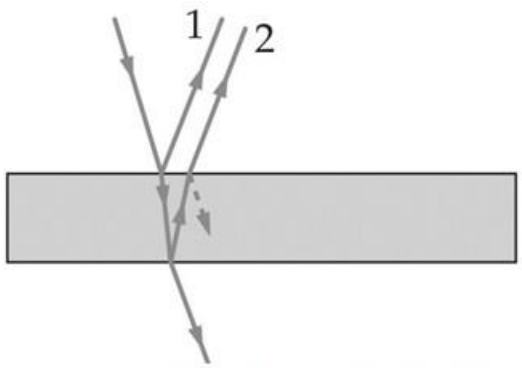


Figure A1-1. Interference in a thin film.

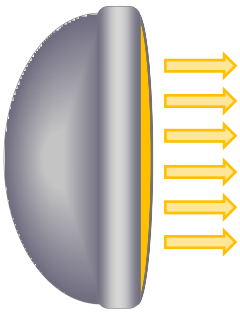


Figure A1-2. An extended light source emits light from a finite area, unlike a point source.

$$\delta = \frac{2\pi}{\lambda} \cdot 2nd \cdot \cos\theta, \qquad (1)$$

where λ is the wavelength in vacuum, d the distance between the plates separated by a medium with refractive index n and θ is the angle to the optical axis (Figure A1-3).

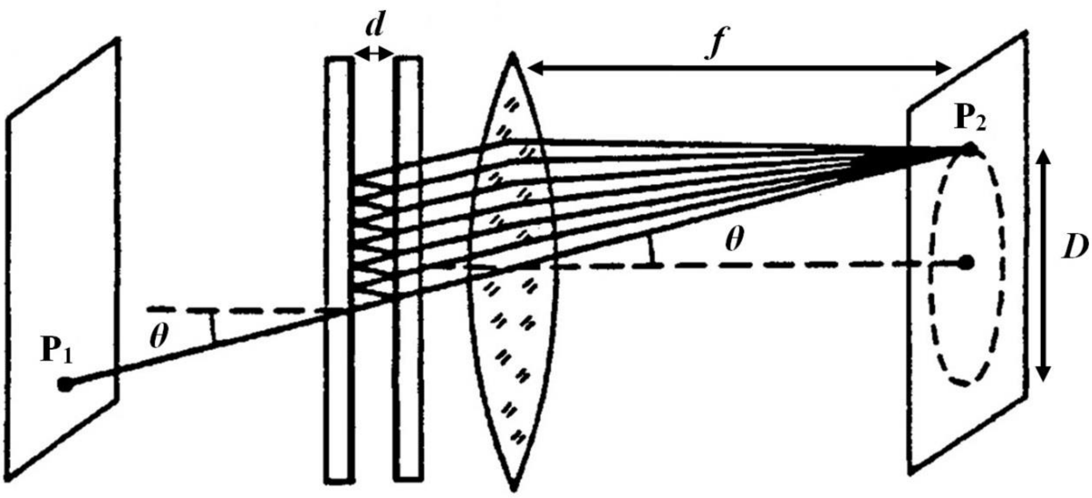


Figure A1-3. Light path through a Fabry-Perot-interferometer.

The condition for constructive interference in point P_2 is:

$$2nd \cdot \cos\theta = m \cdot \lambda \tag{2}$$

In an extended light source all rays that are emitted at an angle of θ will contribute to the ring pattern at the angle of θ to the optical axis. Different orders (i.e. different values of m) will appear with different diameters, as shown in Figure A1-4a.

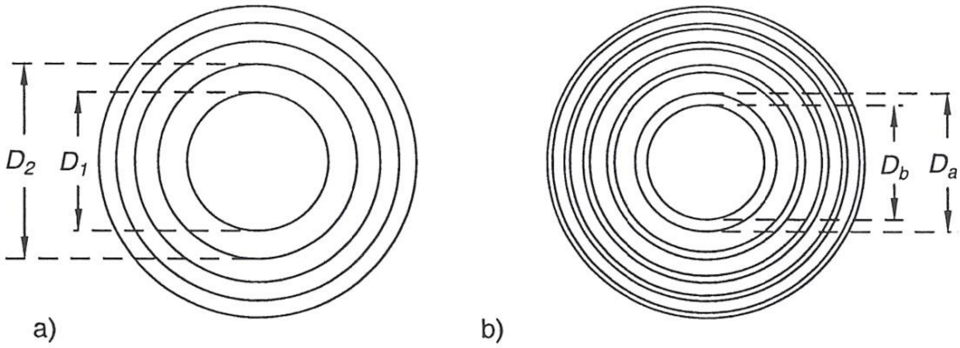


Figure A1-4. a) Interference pattern for a monochromatic light source. b) For a light source with two very close wavelengths.

Transmission and the free spectral range.

The theory of multiple beam interference shows that the transmitted intensity I_t through a Fabry-Perot interferometer is given by the so-called Airy function.

$$I_t = I_0 \cdot A(\delta) = I_0 \cdot \frac{1}{1 + \frac{4R}{(1-R)^2} \cdot \sin^2 \frac{\delta}{2}}, \tag{3}$$

where R = is the reflectance of the surfaces and δ is the phase difference (1).

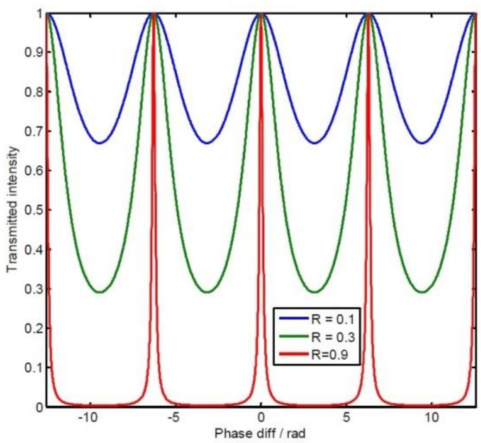


Figure A1-5. The Airy function.

The Airy function is illustrated in Figure A1-5, for different values of the reflectance R. We note both from the figure and from (3) that the function is periodic, with a period of 2π. This period, expressed in any parameter, is called the *free spectral range* (fsr), and is an important parameter since it represents the range over which the interferometer is useful. For example, two wavelengths differing by $\Delta\lambda_{fsr}$ will be imaged on the same ring and hence impossible to distinguish. This is refered to as overlapping orders.

To determine the free spectral range in any parameter other than phase we simply differentiate (1) and use $\Delta\delta_{fsr} = 2\pi$. For example, $\Delta\lambda_{fsr}$ is obtained from

$$\frac{\partial\delta}{\partial\lambda} = (-)\frac{1}{\lambda^2} 4\pi nd \cos(\theta).$$

With $\partial\delta = \Delta\delta_{fsr} = 2\pi$ and $\partial\lambda \approx \Delta\lambda_{fsr}$

$$\Delta\lambda_{fsr} = \frac{\Delta\delta_{fsr} \cdot \lambda^2}{4\pi nd \cos(\theta)} = \frac{\lambda^2}{2nd \cos(\theta)} \approx \frac{\lambda^2}{2nd} \text{ for small angels } \theta. \tag{4}$$

We may obtain the free spectral range in wavenumbers in the same way:

$$\Delta\sigma_{\mathrm{fsr}} = \frac{1}{2dn} \tag{5}$$

Example 1: Free spectral range

Consider the central ring ($\theta = 0$) in a Fabry-Perot interferometer in air ($n = 1$) with 3 mm between the reflecting surfaces (d) and a wavelength of 500 nm. Then $\Delta\lambda_{\mathrm{fsr}} = 0.0417$ nm and $\Delta\sigma_{\mathrm{fsr}} = 1.67$ cm^{-1}. This means that the interferometer can only be used between 500.0000 and 500.0417 nm before the orders start to overlap.

From Example 1 it is clear that such an instrument is not suitable to study large wavelength intervals! On the other hand, we show below that the instrument has a very high resolving power within its useful range.

Smallest detectable wavelength difference $(\Delta\lambda)_{\mathrm{min}}$.

According to the so-called Rayleigh criterion two equally strong lines are said to be resolved if their wavelength difference $(\Delta\lambda)_{\mathrm{min}}$ at least equals the full width at half maximum (FWHM) of the line profiles. This is illustrated in Figure A1-6. To calculate $(\Delta\lambda)_{\mathrm{min}}$ for a Fabry-Perot interferometer, we start by calculating the FWHM in phase of the Airy function.

$$A(\delta) = \frac{1}{2} \Leftrightarrow \frac{1}{1 + \frac{4R}{(1-R)^2} \cdot \sin^2 \frac{\delta}{2}} = \frac{1}{2}$$

$$\Leftrightarrow$$

$$\delta = \pm \frac{1-R}{\sqrt{R}} \text{ and } (\Delta\delta)_{\mathrm{min}} = 2 \cdot \frac{1-R}{\sqrt{R}}$$

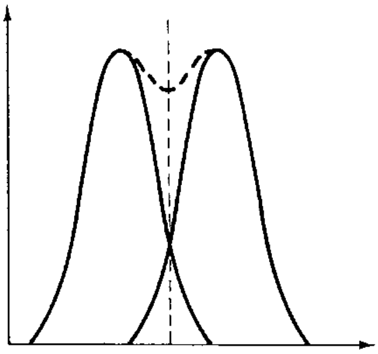


Figure A1-6. *Rayleigh criteria*

To express the width in wavelength we proceed as above and differentiate:

$$\frac{\partial\delta}{\partial\lambda} = (-)\frac{1}{\lambda^2} 4\pi nd \cos(\theta) .$$

With $\partial\delta = (\Delta\delta)_{\mathrm{min}}$ and $\partial\lambda = (\Delta\lambda)_{\mathrm{min}}$ we obtain for small angles θ:

$$(\Delta\lambda)_{\mathrm{min}} = \frac{(\Delta\delta)_{\mathrm{min}} \cdot \lambda^2}{4\pi nd \cos(\theta)} = 2 \cdot \frac{1-R}{\sqrt{R}} \cdot \frac{\lambda^2}{4\pi nd \cos(\theta)} \approx \frac{1-R}{\sqrt{R}} \cdot \frac{\lambda^2}{2\pi nd}$$

Example 2. Smallest detectable wavelength difference.

We continue example 1 above and assume $R = 0.9$. Then $(\Delta\lambda)_{\mathrm{min}} = 0.00139$ nm, giving a resolving power of 357000. Thus very small wavelength differences will give rise to clearly separated and measurable rings.

Wavenumber differences from a measured Fabry-Perot ring system.

From Figure A1-3 we obtain the following relation between θ, the focal length f of the lens, and the diameter D of a circle:

$$\cos\theta = f/\sqrt{f^2+R^2} = (1+R^2/f^2)^{-1/2} \approx 1-\frac{1}{2}\frac{R^2}{f^2} = 1-\frac{D^2}{8f^2}$$

If we combine this with the condition for constructive interference (2) rewritten in terms of wavenumber $\sigma = 1/\lambda$, we obtain:

$$m = 2d\sigma(1-D^2/8f^2)\ .$$

This expression shows that a larger ring corresponds to a smaller m, i.e. a smaller path difference $m\lambda$ between adjacent beams. We can estimate the magnitude of m: Suppose we have a maximum for $\theta = 0$, i.e. $D = 0$, and that we have $d = 3$ mm, which is the distance we use in this experiment. This gives for red light ($\lambda \approx 600$ nm or $\sigma \approx 16\,000$ cm^{-1}) $m \approx 9600$.

Two adjacent spectral lines, e.g. two Zeeman components, give rise to two systems of close lying circles, as seen in Figure A1-4b. If we want to determine the wavenumber difference between two lines in the same interference order m with the wave numbers σ_a and σ_b we get the equations:

$$m = 2d\sigma_a(1-D_a^2/8f^2)$$

$$m = 2d\sigma_b(1-D_b^2/8f^2)$$

We can eliminate m and d and get the equation:

$$\sigma_a(1-D_a^2/8f^2) = \sigma_b(1-D_b^2/8f^2) \qquad (6)$$

D_a and D_b can be measured. The focal length f cannot be measured with sufficient accuracy, but it can be eliminated in the following way. Measure the diameters of the circles of two adjacent orders for the same line σ, e.g. the line we are investigating with no magnetic field (Figure A1-3a). The two circles have the diameters D_1 and D_2 and the orders m and m-1. We get the relations

$$m = 2d\sigma(1-D_1^2/8f^2)$$

$$m-1 = 2d\sigma(1-D_2^2/8f^2)$$

This can be transformed into $2d\sigma(D_2^2-D_1^2) = 8f^2$. Inserting this into (6) and using $\sigma \approx \sigma_a \approx \sigma_b$ gives finally

$$\boxed{\Delta\sigma = \sigma_a - \sigma_b = \frac{1}{2d}\cdot\frac{D_a^2-D_b^2}{D_2^2-D_1^2}\,.} \qquad (7)$$

D_a and D_b are in our case the diameters of the circles from two Zeeman components in a certain order. In practice we use the highest order, i.e. the innermost circles. D_1 and D_2 are the diameters of two circles from the same line - with no magnetic field - in different orders. The distance $d = 3$ mm for the instrument from Phywe (the same instrument that has 3A as its maximum magnetic current) and 3.085 mm in the other interferometers.

Appendix 2 Polarized light

Light can be thought of as an electromagnetic wave consisting of an electric and a magnetic field. The polarization describes how the electrical component of the light behaves both in space and time. If light is polarized, it means that at a given point *z* both the amplitude and the direction of the electric field vary in a regular and predictable way, as opposed to the random variations of unpolarized light.

The superposition principle gives us a convenient way to describe polarized light as a sum of two perpendicular, linearly polarized (along the x- and y-axes) waves with relative phase δ.

$$\overline{E}(x,y,z,t) = E_{0x} \cdot \overline{e}_x \cdot \sin(kz - \omega t + \delta) + E_{0y} \cdot \overline{e}_y \cdot \sin(kz - \omega t)$$

Figure A2-1 shows the resulting polarization for different values of δ for the special case $E_{0x} = E_{0y}$. Note that circularly polarized light only arises when $\delta = (2m + 1)\cdot\pi/2$ <u>and</u> $E_{0x} = E_{0y}$.

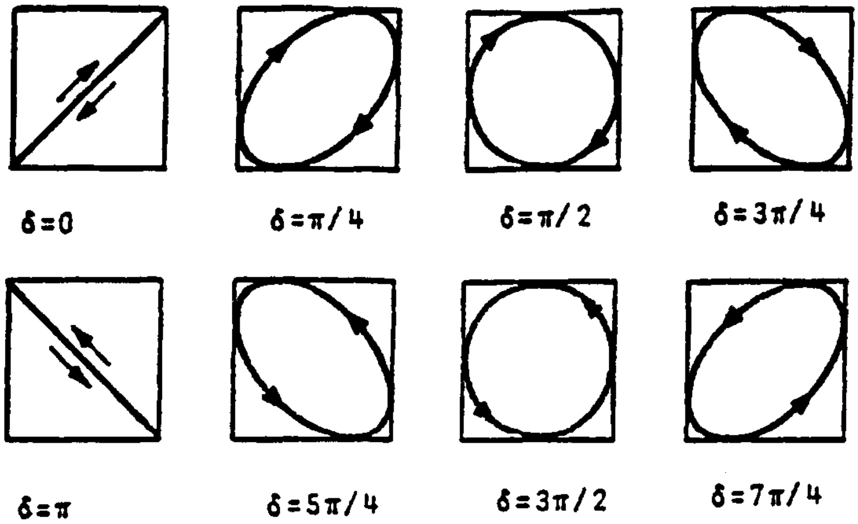


Figure A2-1. Polarization states for different δ when $E_{0x} = E_{0y}$.

Components that change the state of polarization

When light enters a plate of quartz (SiO_2), or other so-called anisotropic materials, it is transmitted as <u>two</u> perpendicular linearly polarized waves (called the ordinary and extra ordinary wave, respectively) that travels with different speeds, i.e. the material has two different indices of refraction n_o and n_e. After passing the plate, with thickness *d*, the two waves have therefore traveled a different optical path length

$$\Delta = d \cdot n_e - d \cdot n_o = d \cdot \Delta n$$

This results in a phase difference δ between the two waves.

$$\delta = \frac{2\pi}{\lambda} \cdot d \cdot \Delta n$$

After the passage the observable light is a superposition of the two waves. In this configuration the plate is called a *retarder plate*. If the incoming light is linearly polarized as in Figure A2-2, i.e. the two oscillations are in phase, the retarder plate can transform this into any desired state of polarization depending on its optical properties. We note, in particular, the following special cases:

$\delta = 2m\cdot\pi$	Full wave plate. No change of the polarization state.
$\delta = (2m+1)\cdot\pi$	Half wave plate. The light is still linear but the plane of oscillation has been rotated so that the new plane of oscillation is orthogonal to the old.
$\delta = (2m+1)\cdot\pi/2$	Quarter wave plate. If the incoming linear light oscillates at 45º to the optical axis of the plate, the amplitude of the two internal waves will be equal and the emerging light will be circularly polarized. <u>Note that the reverse is also true, i.e. if circular light enters then the outgoing light will be linear. This will be used in the lab to prove that the longitudinal Zeeman light is circularly polarized.</u>

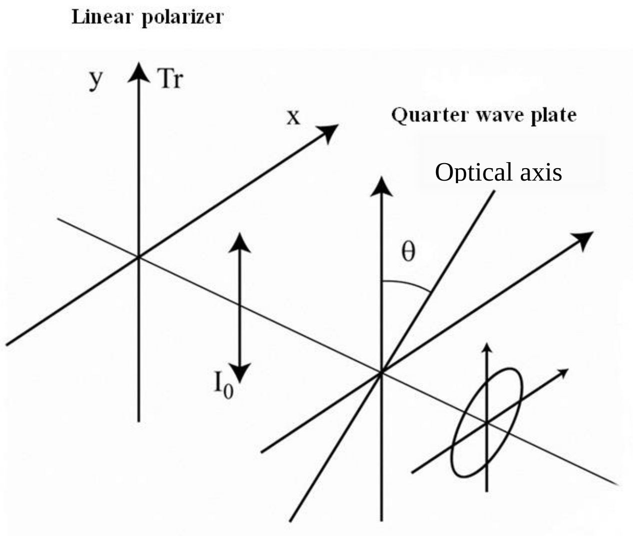


Figure A2-2. A quarter wave plate (λ/4-plate) converts linear light to circular or vise versa.