THE ZEEMAN EFFECT 180206

Introduction

Quantum mechanics tells us that angular momenta, $\bar{J}$, are quantized both regarding magnitude, $J^2 = \hbar^2 J(J+1)$, and direction, $J_z = M_J \cdot \hbar$, as illustrated in Figure 1. The directional quantization is normally not noticeable in the energy level structure in an atom since the "z"-direction is an arbitrary direction (isotropic space), and all states that differ only in the value of M_J have the same energy. The level is said to be $2J + 1$ fold degenerate. However, if the atom is placed in an external magnetic field a specific direction is created, and the different M_J - states obtain a small additional energy:
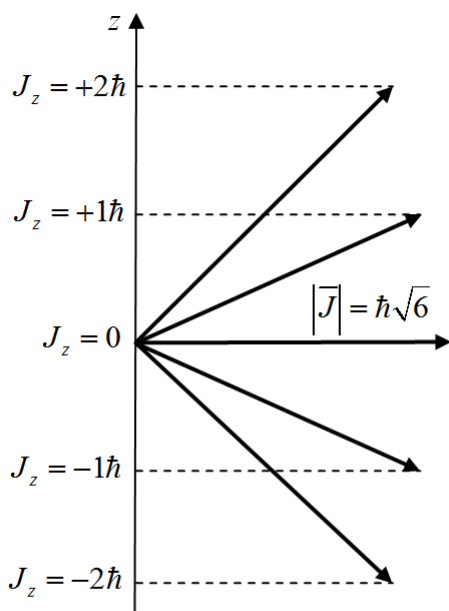
$$E_{mag} = g_J \mu_B B M_J .$$

Here g_J is the Landé factor for the level. Furthermore, light emitted from an excited atom becomes polarized with an oscillation mode that depends on how the M_J quantum numbers change as well as the direction of observation relative to the external magnetic field. Viewed perpendicular to the B-field light is linearly polarized; oscillating parallel to the field for $\Delta M_J = 0$ transitions and perpendicular for $\Delta M_J = \pm 1$. Viewed along the field direction $\Delta M_J = \pm 1$ transitions give rise to circularly polarized light.

Figure 1. Vector model picture of a quantized angular momentum with $J = 2$.

Preparation

Study carefully the theory of the Zeeman effect in e.g. Thorne *et al*, Spectrophysics[1], Ch. 3.9.1 or Foot, Atomic Physics[2], Ch. 1.8 and 5.5. Since the Zeeman splitting of spectral lines is small, we need an instrument with very high spectral resolution to measure it. One such instrument is the Fabry-Perot interferometer described in Spectrophysics 13.1 - 13.4, Pedrotti *et al*, Introduction to Optics[3] and also briefly in Appendix 1, below. You must study in detail (at least) the last section of Appendix 1, where the evaluation of the experiment is described. Recapitulate the description, production and manipulation of polarized light. This is very briefly outlined in Appendix 2

[1]   A. Thorn, U. Litzén and Se. Johansson, Spectrophysics, Springer

[2]   C. Foot, Atomic Physics, Oxford master series in Physics

[3]   F. L. Pedrotti, L. M. Pedrotti and L. S. Pedrotti, Introduction to Optics, Pearson

Preparatory exercises (Serious attempts must be made on <u>all</u> exercises)

1. The ground configuration in neutral Cd is $5s^2$ and the first excited configuration is 5s5p. In the experiment you will, among other lines, see the transition 5s5p ^{3}P - 5s6s ^{3}S.

   a) Give the *LS* notation for the possible transitions between these terms.

   b) You will find a green (λ = 508.582 nm), a turquoise (λ = 479.992 nm) and a blue (λ = 467.816 nm) line. Which of the transitions above correspond to the different colors?

2. **This exercise is essential to the lab and a solution must be presented before you are allowed to continue.**

   In the experiment you will study the transitions 5s5p ^{1}P$_1$ - 5s5d ^{1}D$_2$, 5s5p ^{3}P$_2$ - 5s6s ^{3}S$_1$, 5s5p ^{3}P$_1$ - 5s6s ^{3}S$_1$ and 5s5p ^{3}P$_0$ - 5s6s ^{3}S$_1$ in a weak magnetic field.

   a) Derive the Landé *g* factor assuming *LS* coupling for the levels involved in the four transitions.

   b) Draw <u>large and nice</u> diagrams showing the different Zeeman components that each of the 4 <u>lines</u> (not levels) split into in the magnetic field in the manner of Figure 3.16 in Spectrophysics or Figure 5.13 in Atomic Physics. Thus, choose a relative energy scale, with zero at the energy of the transition without magnetic field, and show the splittings in units of $\mu_B B$ along the x – axis. Let all Zeeman components have the same intensity.

   c) What is the state of polarization of each of the components?

   d) Which components do you expect to see in a direction parallel to the magnetic field?

3. Let $B = 0.5$ T. How large is the smallest splitting between the components derived above?

   a) Expressed in eV

   b) Expressed in cm^{-1}

   c) Expressed in nm

4. Use Appendix 1 to answer the following. A Fabry-Perot interferometer operating in air have mirror surfaces with a reflectance of $R = 0.85$ and separated by 3.085 mm. We use a light source with a wavelength of 500 nm.

   a) What is the free spectral range expressed in cm^{-1} and in nm.

   b) What is the line width expressed in cm^{-1} and nm.

   c) Does the size of the rings increase or decrease in higher spectral orders?

5. Use Appendix 2 to answer the following. What is the polarization of light when the electric field is described by the expressions below?

   a) $\bar{E} = E_0 \cdot (\bar{e}_x \cdot \sin(kz - \omega t) + \bar{e}_y \cdot \cos(kz - \omega t))$

   b) $\bar{E} = 5 \cdot \bar{e}_x \cdot \sin(kz - \omega t + \pi/2) + 3 \cdot \bar{e}_y \cdot \sin(kz - \omega t)$

Experiments

Setup

Neutral cadmium atoms have the ground configuration $5s^2$ and the normal system of excited levels $5sn\ell$. In the experiment you are going to study the 4 lines

$$5s5p\ ^1P_1 - 5s5d\ ^1D_2 \quad 643.8\ \text{nm (red)}$$

$$5s5p\ ^3P_2 - 5s6s\ ^3S_1 \quad 508.6\ \text{nm (green)}$$

$$5s5p\ ^3P_1 - 5s6s\ ^3S_1 \quad 480.0\ \text{nm (turquoise)}$$

$$5s5p\ ^3P_0 - 5s6s\ ^3S_1 \quad 467.8\ \text{nm (blue)}$$

The cadmium spectrum is emitted from a spectral lamp placed between the poles of an electromagnet, where the field is directly proportional to the current. Light then passes through a Fabry-Perot interferometer and the interference pattern is studied with a CCD camera. The set up is shown in Figure 2. The lenses have focal lengths 50, 300 and 50 mm, respectively, countered from the magnet, and the approximate distances, in cm, from the Cd-lamp are given below the components.
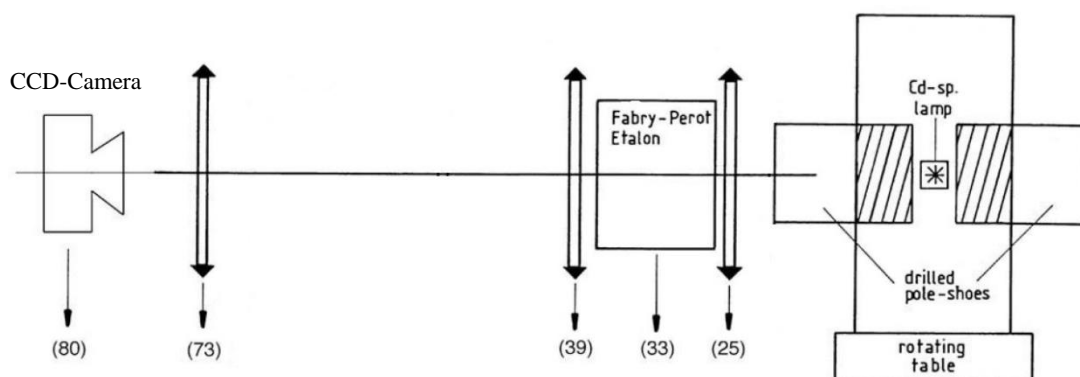

Figure 2. The experimental set up.

The red and green spectral lines can be software selected using the RGB settings of the camera itself. To isolate the closely spaced blue and turquoise lines we need external narrow band interference filters. Figure 3a shows the transmission curves for the filters.
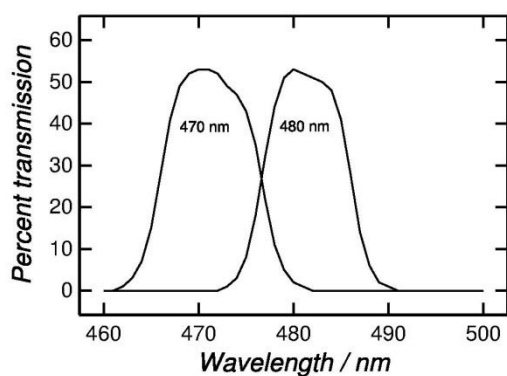

Figure 3a. Transmission curves for the interference filters used. Data from Thorlabs[4].

Figure 3b. Actual retardance for the plate that is λ/4 at 488 nm. Data from Thorlabs[4].

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[4]   http://www.thorlabs.com/

The polarization of the Zeeman components of light emitted perpendicularly and parallel to the magnetic field is investigated using a linear polarizer and a zero-order (achromatic) $\lambda/4$ plate. The actual, slightly wavelength dependent, retardance of the $\lambda/4$ plate is given in Figure 3b.

Qualitative studies - actually the most important part!

Start by viewing the Fabry-Perot interference patterns directly by putting your eye immediately behind the last lens. Use the different filters and different magnetic fields.
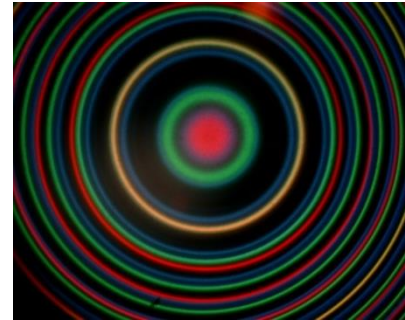

Figure 4. Fabry-Perot pattern with no filters or magnetic field.

Then carefully adjust and center the camera on the pattern and capture images with the provided software. Learn how to use the software. First to record the images with different exposures, RGB settings, noise reduction and so on, to obtain the best images. The supervisor will help you if need be.

Select each of the four transitions in turn. Observe the splittings in both transverse and longitudinal direction, as you vary the magnetic field. Do your observations agree with the predictions in the preparatory exercise 2? Insert the polarization components and verify again the predictions in exercise 2.

Quantitative studies.

Perform a set of measurements in the longitudinal direction for both the red and the blue line. Or use the transverse direction but then with a vertical polarizer inserted to isolate the σ-transitions. Use suitable magnetic currents in steps of 0.5 A between 0 and, at most, 5A and capture the ring pattern for each setting to a file.

As a preparation for the quantitative analysis according to eq. (7) in Appendix 1, use the matlab code to determine the diameter of the observed rings for each magnetic current, as illustrated in Figure 5. The supervisor will help you if need be.
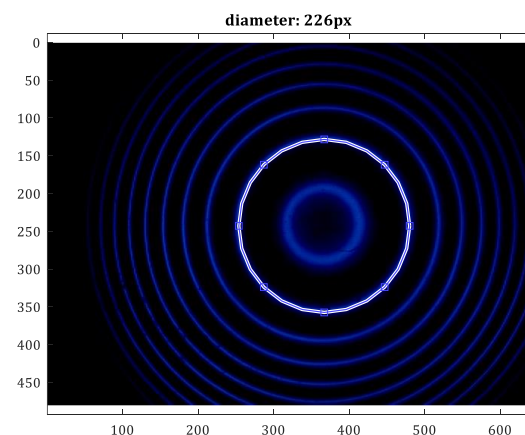

Figure 5. Illustration on how to use the provided matlab code to analyze the ring patterns.

Use eq. (7) in Appendix 1 and your measured diameters and plot, in the same diagram, the Zeeman splitting, $\Delta\sigma$, as a function of the current through the magnet for the red and blue line. The magnetic field is directly proportional to the current, as shown in Figure 6 for currents up to 5 A. Verify that the Zeeman splitting is, indeed, directly proportional to the magnetic field. To compare the numerical value of the splittings to the theoretical prediction you need the approximate proportionality constants which are 0.1 T/A for the Phywe magnets and 0.058 T/A for the "large magnet".
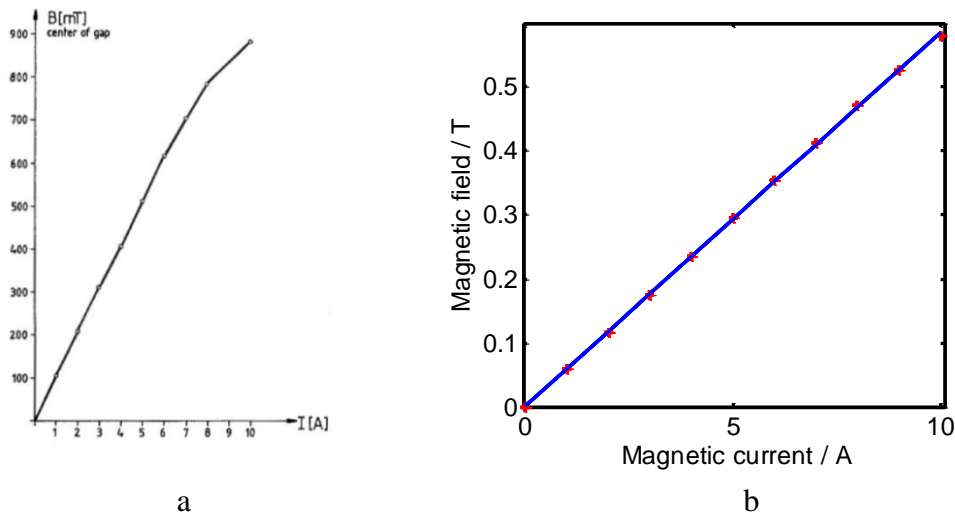

a	b

Figure 6. Magnetic field as a function of current for the a) Phywe magnets and b) "Large magnet".

To eliminate also the need to accurately know the thickness of the spacer between the plates in eq. (7), show that the ratio of the slopes of the red and blue lines is accurately predicted by the quantum numbers and g_J - factors involved, see preparatory exercise 2.

Appendix 1:

The Fabry-Perot interferometer, free spectral range, line width and data reduction for the Zeeman lab.

General

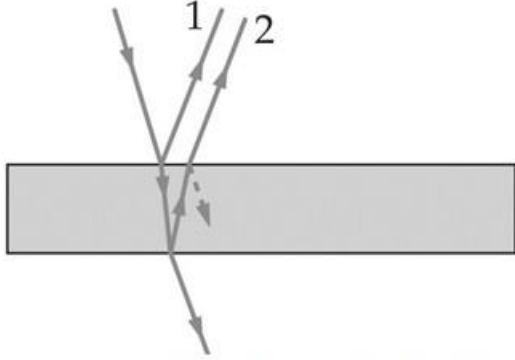You have most likely discussed the phenomenon of interference in thin films, shown in

Figure A1-1. Interference in a thin film.

Figure A1-1, in some earlier course. In many cases the reflectance of the surfaces is low and one need only to take the first two rays into account. However, if the reflectance is high we must sum the contribution from (very) many rays, a phenomenon called multiple beam interference. This is the basis for the Fabry-Perot interferometer. A typical interferometer is shown schematically in Figure A1-2. Two plane glass plates with highly reflecting surfaces facing each other are separated by a distance d. Light from an extended source pass through the system, as in Figure A1-2. The light is reflected many times between the plates and at each reflection a small part is transmitted. Each transmitted wave acquires a phase difference of δ relative to the previous one due to the extra "round trip" between the plates.

$$\delta = \frac{2\pi}{\lambda} \cdot 2nd \cdot \cos\theta, \qquad (1)$$

where: λ is the wavelength in vacuum, d the distance between the plates separated by a medium with refractive index n and θ is the angle to the centrum axis (Figure A1-2).
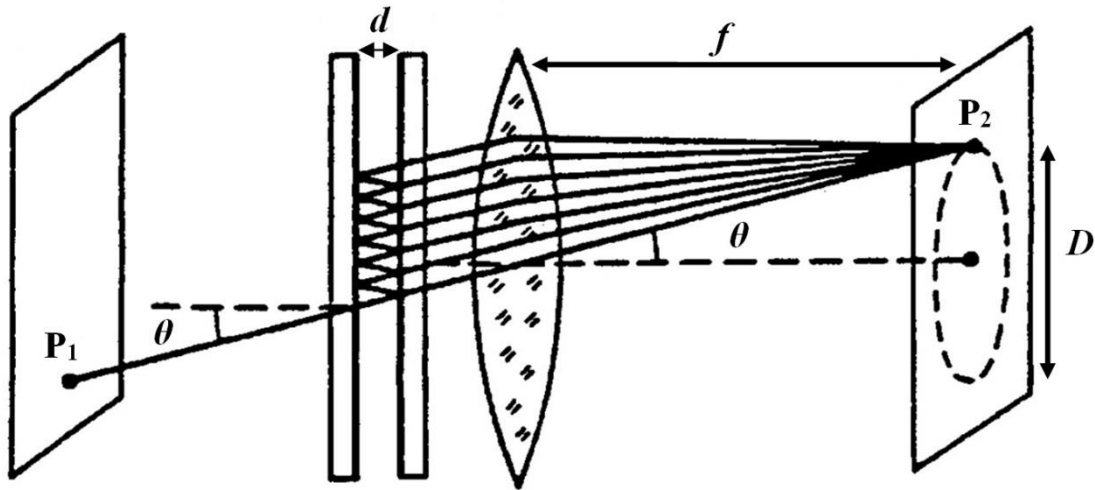

Figure A1-2. Light path through a Fabry-Perot-interferometer.

The condition for constructive interference in point P_2 is:

$$2nd \cdot \cos\theta = m \cdot \lambda \qquad (2)$$

In an extended light source all points P_1 on a circle will enter with the same angle θ and

hence the interference pattern will consist of concentric circles for different values of *m*, as shown in Figure A1-3.
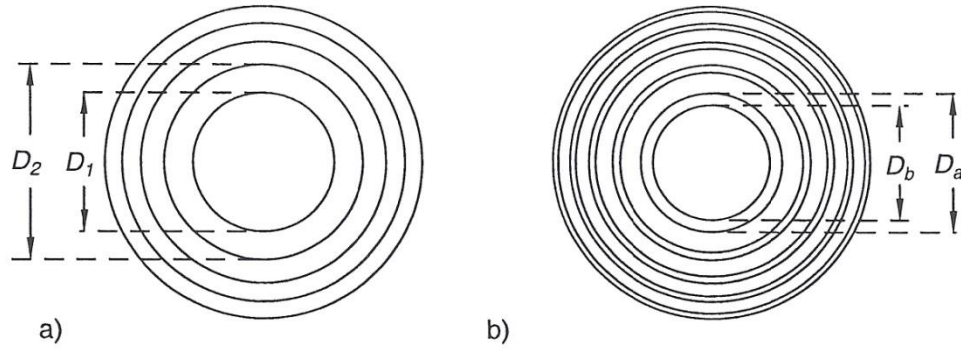

Figure A1-3**.** a) Interference pattern for a monochromatic light source. b) For a light source with two very close wavelengths.

Transmission and the free spectral range.

The theory of multiple beam interference shows that the transmitted intensity I_t through a Fabry-Perot interferometer is given by the so-called Airy function.

$$I_t = I_0 \cdot A(\delta) = I_0 \cdot \frac{1}{1 + \dfrac{4R}{(1-R)^2} \cdot \sin^2 \dfrac{\delta}{2}}, \qquad (3)$$

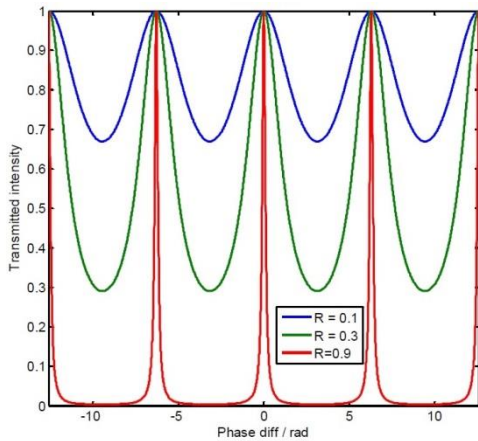where $R = $ is the reflectance of the surfaces and δ is the phase difference (1).

The Airy function is illustrated in Figure A1-4, for different values of the reflectance *R*. We note both from the figure and from (3) that the function is periodic, with a period of 2π. This period, expressed in any parameter, is called the *free spectral range* (fsr), and is an important parameter since it represents the range over which the interferometer is useful. For example, two wavelengths differing by $\Delta\lambda_{\text{fsr}}$ will be imaged on the same ring and hence impossible to distinguish. This is refered to as overlapping orders.

Figure A1-4. The Airy function

To determine the free spectral range in any parameter other than phase we simply differentiate (1) and use $\Delta\delta_{\text{fsr}} = 2\pi$. For example, $\Delta\lambda_{\text{fsr}}$ is obtained from

$$\frac{\partial \delta}{\partial \lambda} = (-) \frac{1}{\lambda^2} 4\pi n d \cos(\theta).$$

With $\partial\delta = \Delta\delta_{\text{fsr}} = 2\pi$ and $\partial\lambda \approx \Delta\lambda_{\text{fsr}}$

$$\Delta\lambda_{\text{fsr}} = \frac{\Delta\delta_{fsr} \cdot \lambda^2}{4\pi nd\cos(\theta)} = \frac{\lambda^2}{2nd\cos(\theta)} \approx \frac{\lambda^2}{2nd} \text{ for small angels } \theta. \qquad (4)$$

We may obtain the free spectral range in wavenumbers in the same way

$$\Delta\sigma_{\text{fsr}} = \frac{1}{2dn} \qquad (5)$$

Example 1: Free spectral range

Consider the central ring ($\theta = 0$) in a Fabry-Perot interferometer in air ($n = 1$) with 3 mm between the reflecting surfaces (d) and a wavelength of 500 nm. Then $\Delta\lambda_{\text{fsr}} = 0.0417$ nm and $\Delta\sigma_{\text{fsr}} = 1.67$ cm^{-1}. This means that the interferometer can only be used between 500.0000 and 500.0417 nm before the orders start to overlap.

It is clear that such an instrument is not suitable to study large wavelength intervals! On the other hand, we show below that the instrument has a very high resolving power within its useful range.

Smallest detectable wavelength difference ($\Delta\lambda$)$_{\text{min}}$.

According to the so-called Rayleigh criteria two equally strong lines are said to be resolved if their wavelength difference $(\Delta\lambda)_{\text{min}}$ at least equals the full width at half maximum (FWHM) of the line profiles. This is illustrated in Figure A1-5. To calculate $(\Delta\lambda)_{\text{min}}$ for a Fabry-Perot interferometer, we start by calculating the FWHM in phase of the Airy function.

$$A(\delta) = \frac{1}{2} \Leftrightarrow \frac{1}{1 + \frac{4R}{(1-R)^2} \cdot \sin^2\frac{\delta}{2}} = \frac{1}{2}$$

$$\Leftrightarrow$$

$$\delta = \pm\frac{1-R}{\sqrt{R}} \text{ and } (\Delta\delta)_{\text{min}} = 2 \cdot \frac{1-R}{\sqrt{R}}$$
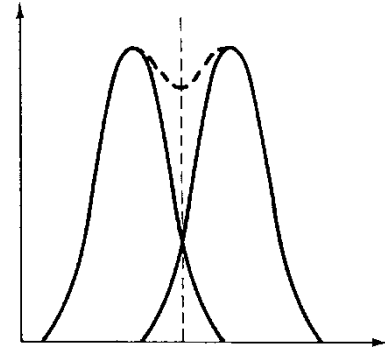
Figure A1-5. *Rayleigh criteria*

To express the width in wavelength we proceed as above and differentiate:

$$\frac{\partial\delta}{\partial\lambda} = (-)\frac{1}{\lambda^2} 4\pi nd\cos(\theta).$$

With $\partial\delta = (\Delta\delta)_{\text{min}}$ and $\partial\lambda = (\Delta\lambda)_{\text{min}}$ we obtain for small angles θ:

$$(\Delta\lambda)_{\text{min}} = \frac{(\Delta\delta)_{\text{min}} \cdot \lambda^2}{4\pi nd\cos(\theta)} = 2 \cdot \frac{1-R}{\sqrt{R}} \cdot \frac{\lambda^2}{4\pi nd\cos(\theta)} \approx \frac{1-R}{\sqrt{R}} \cdot \frac{\lambda^2}{2\pi nd}$$

Example 2. Smallest detectable wavelength difference.

We continue example 1 above and assume $R = 0.9$. Then $(\Delta\lambda)_{\text{min}} = 0.00139$ nm, giving a resolving power of 357000. Thus very small wavelength differences will give rise to clearly separated and measurable rings.

Wavenumber differences from a measured Fabry-Perot ring system.

From Figure A1-2 we obtain the following relation between θ, the focal length f of the lens, and the diameter D of a circle:

$$\cos\theta = f / \sqrt{f^2 + R^2} = (1 + R^2 / f^2)^{-1/2} \approx 1 - \frac{1}{2}\frac{R^2}{f^2} = 1 - \frac{D^2}{8f^2}$$

If we combine this with the condition for constructive interference (2) rewritten in terms of wavenumber $\sigma = 1/\lambda$, we obtain:

$$m = 2d\sigma(1 - D^2 / 8f^2) .$$

This expression shows that a larger ring corresponds to a smaller m, i.e. a smaller path difference $m\lambda$ between adjacent beams. We can estimate the magnitude of m: Suppose we have a maximum for $\theta = 0$, i.e. $D = 0$, and that we have $d = 3$ mm, which is the distance we use in this experiment. This gives for red light ($\lambda \approx 600$ nm or $\sigma \approx 16\,000$ cm^{-1}) $m \approx 9600$.

Two adjacent spectral lines, e.g. two Zeeman components, give rise to two systems of close lying circles, as seen in Figure A1-3b. If we want to determine the wavenumber difference between two lines in the same interference order m with the wave numbers σ_a and σ_b we get the equations:

$$m = 2d\sigma_a(1 - D_a^2 / 8f^2)$$
$$m = 2d\sigma_b(1 - D_b^2 / 8f^2)$$

We can eliminate m and d and get the equation:

$$\sigma_a(1 - D_a^2 / 8f^2) = \sigma_b(1 - D_b^2 / 8f^2) \tag{6}$$

D_a and D_b can be measured. The focal length f cannot be measured with sufficient accuracy, but it can be eliminated in the following way. Measure the diameters of the circles of two adjacent orders for the same line σ, e.g. the line we are investigating with no magnetic field (Figure A1-3a). The two circles have the diameters D_1 and D_2 and the orders m and $m-1$. We get the relations

$$m = 2d\sigma(1 - D_1^2 / 8f^2)$$
$$m - 1 = 2d\sigma(1 - D_2^2 / 8f^2)$$

This can be transformed into $2d\sigma(D_2^2 - D_1^2) = 8f^2$. Inserting this into (6) and using $\sigma \approx \sigma_a \approx \sigma_b$ gives finally

$$\Delta\sigma = \sigma_a - \sigma_b = \frac{1}{2d} \cdot \frac{D_a^2 - D_b^2}{D_2^2 - D_1^2} . \tag{7}$$

D_a and D_b are in our case the diameters of the circles from two Zeeman components in a certain order. In practice we use the highest order, i.e. the innermost circles. D_1 and D_2 are the diameters of two circles from the same line - with no magnetic field - in different orders. The distance $d = 3$ mm for the instrument from Phywe and 3.085 mm in the other interferometers.

Appendix 2 Polarized light

Light can be thought of as an electromagnetic wave consisting of an electric and a magnetic field. Since light interact with matter mainly through the electric part only this is usually discussed. Polarized light means that at a given point z both the amplitude and the direction of the electric field vary in a regular and predictable way, as opposed to the random variations in unpolarized light.

The superposition principle gives us a convenient way to describe polarized light in general as a sum of two perpendicular, linearly polarized waves with different relative phase, δ.

$$\bar{E}(x, y, z, t) = E_{0x} \cdot \bar{e}_x \cdot \sin(kz - \omega t + \delta) + E_{0y} \cdot \bar{e}_y \cdot \sin(kz - \omega t)$$

Figure A2-1 shows the resulting polarization for different values of δ for the special case $E_{0x} = E_{0y}$.
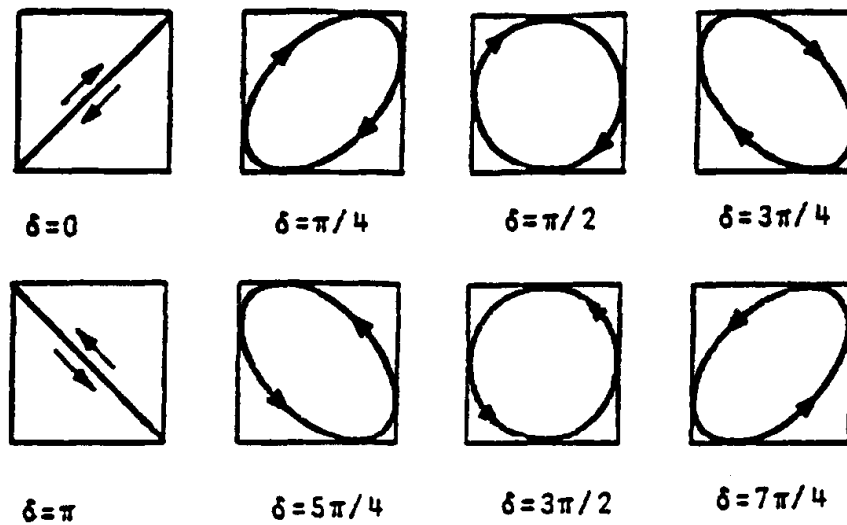

Figure A2-1. Polarization states for different δ, when $E_{0x} = E_{0y}$.

Note the special case of circular light arising when $\delta = (2m + 1) \cdot \pi/2$ <u>and</u> $E_{0x} = E_{0y}$.

Components that change the state of polarization

When light enters a plate of quartz (SiO_2), or other so-called anisotropic materials, it is transmitted as <u>two</u> perpendicular linearly polarized waves (called the ordinary and extra ordinary wave, respectively) that travels with different speeds, i.e. the material has two different index of refraction n_o and n_e. After passing the plate, with thickness d, the two waves have therefore traveled a different optical path length

$$\Delta = d \cdot n_e - d \cdot n_o = d \cdot \Delta n$$

This results in a phase difference δ between the two waves.

$$\delta = \frac{2\pi}{\lambda} \cdot d \cdot \Delta n$$

After the passage the observable light is a superposition of the two waves. In this configuration the plate is called a *retarder plate*. If the incoming light is linearly polarized as in Figure A2-2, i.e. the two oscillations are in phase, the retarder plate can transform this into any desired state of polarization depending on its optical properties. We note, in particular, the following special cases:

$\delta = 2m \cdot \pi$ \qquad Full wave plate. No change of the polarization state

$\delta = (2m +1) \cdot \pi$ \qquad Half wave plate. The light is still linear but the plane of oscillation has been rotated.

$\delta = (2m +1) \cdot \pi/2$ \quad Quarter wave plate. If the incoming linear light oscillates at 45° to the optical axis of the plate, the amplitude of the two internal waves will be equal and the emerging light will be circularly polarized. Note that the reverse is also true, i.e. if circular light enters then the outgoing light will be linear. This will be used in the lab to prove that the longitudinal Zeeman light is circularly polarized
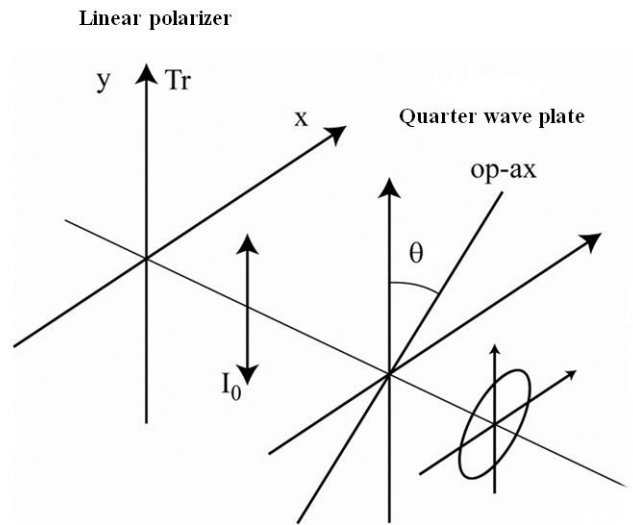

Figure A2-2. A quarter wave plate (λ/4-plate) converts linear light to circular or vise versa.