Quantum Information Lab: Preparatory exercises

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1. The dark state

In a two-color pulse we send in light that targets two different transitions as can be seen in figure 1. The two different parts of the two-color pulse have different frequencies ω_0 and ω_1, they also have different Rabi frequencies Ω_0 and Ω_1 as well as individual phases ϕ_0 and ϕ_1.

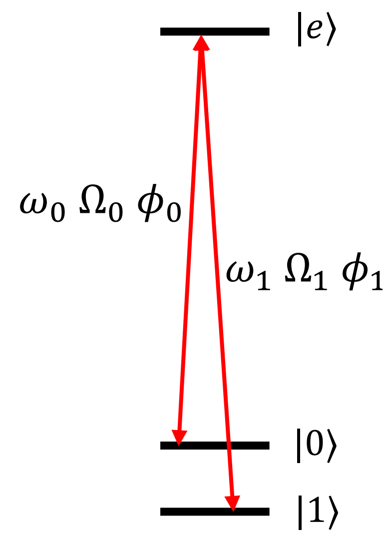

Figure 1) A three level system with two qubit levels $|0\rangle$ and $|1\rangle$, and an excited state $|e\rangle$. Our two-color pulse is targeting the two transitions from the two different ground levels to the excited level.

If we denote the population in each level by c_i where $i = 0, 1$, or e; a simple rate equation can be written for the population in the excited state as:

$$\dot{c}_e = \frac{i\Omega_0}{2} e^{i\phi_0} c_0 + \frac{i\Omega_1}{2} e^{i\phi_1} c_1$$

Now assume that the incoming Rabi frequencies are the same, i.e., $\Omega_0 = \Omega_1 = \Omega_R$ and define $\phi = \phi_1 - \phi_0$ and determine the state that doesn't interact with the excited level, i.e., the state (c_0, c_1) that gives $\dot{c}_e = 0$.

2. A pair of two-color pulses

As described in the lab manual a pair of two-color pulses with the same relative phase factor ϕ between the two pulses within a pair, but a difference in the overall phase θ between the two pairs, can in the $|B\rangle$, $|D\rangle$ basis be described by the operator:

$$U_{TC}^{BD} = \begin{pmatrix} e^{i\theta} & 0 \\ 0 & 1 \end{pmatrix} = e^{i\theta}|B\rangle\langle B| + |D\rangle\langle D|$$

Where we have defined the bright and dark states as:

$$|B\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + e^{-i\phi}|1\rangle\right)$$

$$|D\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - e^{-i\phi}|1\rangle\right)$$

Please rewrite U_{TC}^{BD} in the $|0\rangle$, $|1\rangle$ basis, i.e., U_{TC}^{01}.

3. NOT gate

Use the results from assignment 2 to determine which ϕ and θ gives a NOT gate on the qubit levels $|0\rangle$ and $|1\rangle$ (neglect any global phase factor).

4. Quantum state tomography rotations

First determine for which ϕ the interaction matrix, U_{TC}^{01}, corresponds to a rotation around the x and y axis, respectively (neglect any global phase factor). Then determine which combinations of ϕ and θ are needed to perform the rotations necessary to perform a quantum state tomography in all bases.

5. Superposition state

Choose one of the four superpositions states written below and write out which combination of ϕ and θ would bring a state initially in $|0\rangle$ to that superposition.

$$\Psi_a = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$
$$\Psi_b = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$
$$\Psi_c = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$$
$$\Psi_d = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$$