Quantum Information Lab: A214

Introduction

Quantum information and quantum computing are growing research topics at the moment. At the heart of the interest for these topics lies the huge potential for increased computational power when utilizing quantum phenomena, such as superpositions and entanglement. One of the most famous examples is the Shor's algorithm for prime factorization. The mathematician Peter Shor showed in the nineties that, on a quantum computer, one could perform prime factorization with exponential speedup in comparison with a classical machine, thus transforming a computationally hard problem, to an easily manageable one. As an added flavor to this, prime factorization is the main ingredient in many of today's encryption systems, so with a quantum computer much encrypted information could easily be broken. Quantum mechanics has however also supplied us with a solution to this problem, quantum cryptography. This is a more mature field that has already taken the step from the laboratory environment into commercial products.

In addition to this, classical computers are also becoming smaller and smaller and approaching the point where quantum phenomena cannot be avoided. It is therefore clear that understanding and ultimately being able to control the quantum world will be an important task for the future. In this lab we aim to show the basics of how to measure as well as manipulate a quantum system.

The qubit ion

The ion that we will be using during the lab is Praseodymium (Pr^{3+}). It belongs to the group of the rare-earth ions in the periodic table. Common to all atoms in this group is that their excited states can have long lifetimes and coherence time. The reason for this is outer lying electron orbitals that shield the inner, active one, from charge fluctuations. Figure 1 shows the level structure of the Pr ion, including the splittings between both the different hyperfine levels and the electronic levels. Two of the hyperfine levels in the ground state are chosen to represent the states $|0\rangle$ and $|1\rangle$ of our qubit. Another way of putting it is that our qubit is encoded in the nuclear spin of the atom. The third level is called auxiliary and it is used to store ions that would otherwise disturb the quantum computation process. All operations on the two qubit states, $|0\rangle$ and $|1\rangle$, are done using optical pulses that go via the electronically excited state, $|e\rangle$, as indicated in Figure 1.

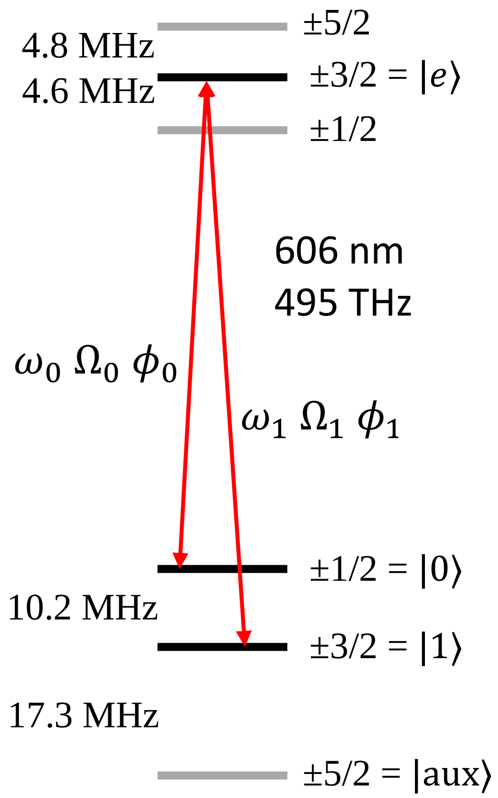

Figure 1) Energy level diagram of the Pr^{3+} ion. Quantum numbers for the nuclear spin Zeeman levels and level splittings are shown to the right and left, respectively.

It will also be important during the lab to know the relative transition strengths of the nine different possible transitions you get between the three ground state levels and the three exited state levels. These values are given in the table below:

As can be seen, the diagonal elements are stronger than the off-diagonal ones, and an atom in the 5/2-level in the excited state has a particularly high chance of being deexcited to the corresponding 5/2-level in the ground state.

g \ e	± 1/2	± 3/2	± 5/2
± 1/2	0.56	0.38	0.06
± 3/2	0.39	0.60	0.01
± 5/2	0.05	0.02	0.93

Table 1) Relative oscillator strengths of the Pr^{3+} ion transitions.

The excited state lifetime of the Pr ions are around 150 µs, and the lifetime of the ground state hyperfine levels are many seconds. The coherence time however, for hyperfine states (the qubit coherence time) is around 500 µs at zero magnetic field (but can be orders of magnitude longer using an appropriately oriented magnetic field).

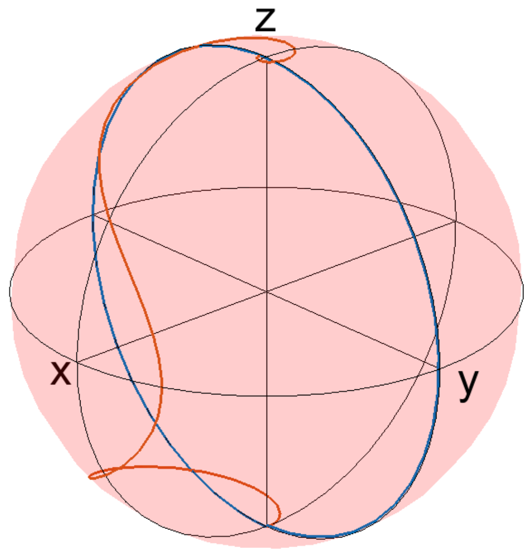

Figure 2) Bloch sphere showing the path of a simple square pulse in blue and the path of the more complicated sechyp pulse in red.

Sechyp pulses

In order to control the qubit and perform gate operations we send in optical light which can be tuned to match any of the nine transitions (from any of the three ground levels to any of the three excited levels). A simple square pulse with a constant Rabi frequency will rotate the state vector in a circle around the Bloch sphere, as can be seen in the blue line in figure 2. Unfortunately, these pulses are neither robust against laser intensity fluctuations nor good at transferring detuned ions. Therefore, more complex pulses are needed.

The sechyp pulse, however, is robust in these situations. Now the Rabi frequency and the light frequency of the incoming pulse are changed according to the following:

$$\Omega_{amp} = \Omega_0 \operatorname{sech}(\beta t)$$
$$f = \mu\beta \tanh(\beta t) \quad (1)$$

Where β and μ are parameters that relate to the frequency chirp, f, and the duration of the pulse and Ω_0 is the maximum Rabi frequency. An example of how the amplitude and the frequency of a sechyp pulse changes with time can be seen in figure 3. Furthermore, the effect of such a sechyp pulse on an atomic 2-level system is seen in the red line in figure 2. As mentioned before, these pulses have the benefit compared to simple square pulses that they are robust against intensity fluctuations of the laser and better at transferring ensembles, consisting of ions that have slightly different frequencies between the upper and lower states, with high fidelity, but only when performing pole to pole transfers.

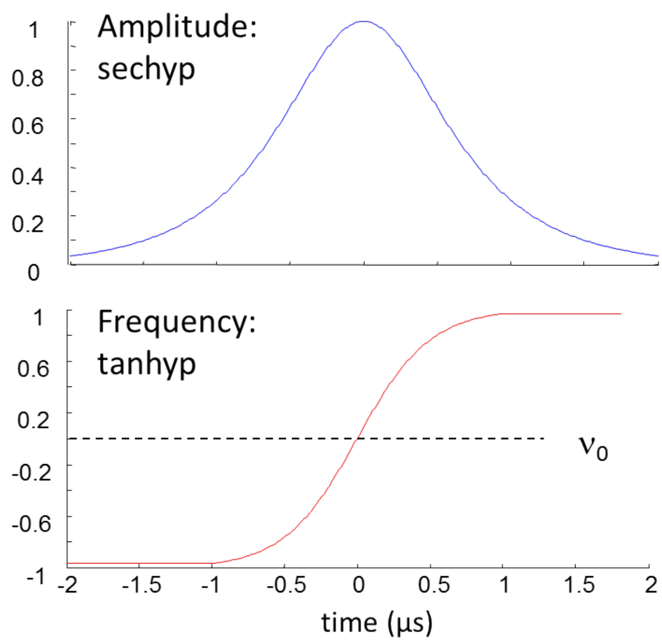

Figure 3) Sechyp amplitude and frequency changes as a function of time.

In order to fully control our system we need to be able to perform more than just pole to pole transfers of our qubit states. Fortunately, since our qubit works with three levels (the two qubit levels $|0\rangle$ and $|1\rangle$, and the excited level $|e\rangle$ which is only used as an aid to perform the qubit operations between the two ground levels), two-color pulses can be used to overcome this issue. These pulses are the topic of the next section.

Two-color pulses: Dark and bright states

Two-color pulses (dark/bright state pulses) are pulses where the incoming electromagnetic field has two frequencies, each resonant with a different transition, for example as represented by the two arrows in figure 1. Each of these two fields can have a different Rabi frequency Ω_i and phase ϕ_i. In order to simplify calculations we assume that the Rabi frequencies are equal $\Omega_0 = \Omega_1$ and we define the relative phase difference as $\phi = \phi_1 - \phi_0$. This can be expressed as two states called bright, $|B\rangle$, and dark, $|D\rangle$, which, respectively, will and will not interact with the light (due to interference of the two light waves with different frequencies);

$$|B\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + e^{-i\phi}|1\rangle\right)$$

$$|D\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - e^{-i\phi}|1\rangle\right)$$

These states will always lie in the equatorial plane and be pointing in opposite directions. The phase difference ϕ determines the angle in the equatorial plane, see figure 4 for an example. Note that this Bloch sphere connects the two qubit levels $|0\rangle$ and $|1\rangle$.

Now, if two sechyp pulses, one at frequency ω_0 and the other at frequency ω_1, are used simultaneously to perform a two-color pole to pole transfer, they will bring the fraction of the wave function in the bright state, $|B\rangle$, to the excited state, whereas the dark state, $|D\rangle$, part of the wave function is left alone since it does not interact with the incoming light. In other words, one two-color sechyp pulse will transfer $|B\rangle \rightarrow |e\rangle$ whilst doing nothing to $|D\rangle$ and any arbitrary state $\alpha|0\rangle + \beta|1\rangle$ can be expressed in the $|B\rangle$, $|D\rangle$ basis.

By using yet another two-color pulse with the same relative phase difference ϕ as before, i.e., it targets the same bright and dark states as the first pulse, but now with an added overall phase factor θ, the path

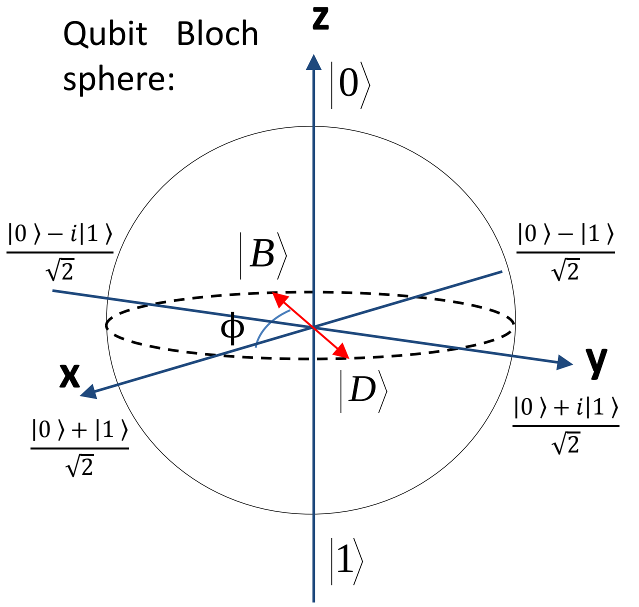

Figure 4) Qubit Bloch sphere with $|0\rangle$ and $|1\rangle$ on the north and south poles, respectively, and also showing an example of $|B\rangle$ and $|D\rangle$ states.

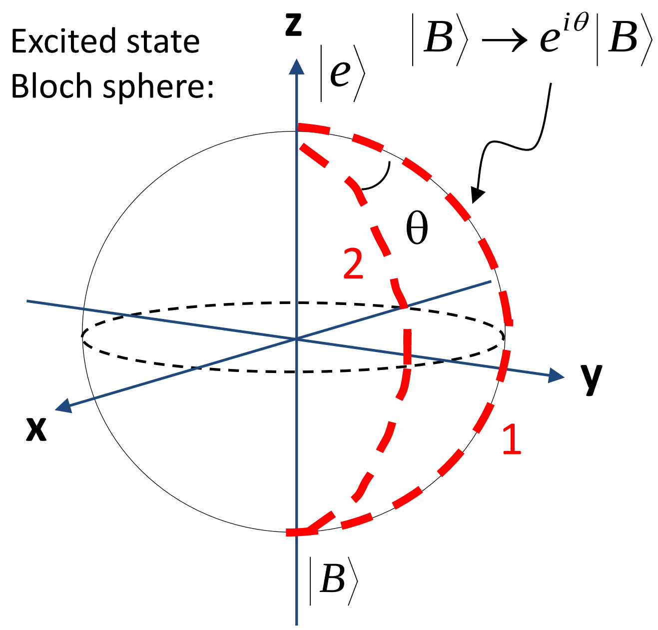

Figure 5) Excited state Bloch sphere with $|e\rangle$ and $|B\rangle$ on the north and south poles, respectively. Shows the path the state vector will travel when subjected to two consecutive two-color pulses with an overall phase difference θ, but the same relative phase difference ϕ between the two pulses within each pair.

taken by the bright state will be as shown in figure 5. Note that this Bloch sphere is going between the bright state $|B\rangle$ and the excited state $|e\rangle$.

As can be seen in the figure, both two-color pulses perform a pole to pole transfer, but along different paths, due to the overall extra phase factor θ. The state will after both pulses end up back in the bright state, but with an added phase $e^{i\theta}$, i.e., the two pulses perform the operation $|B\rangle \rightarrow e^{i\theta}|B\rangle$ and $|D\rangle \rightarrow |D\rangle$ since the dark state is still unaffected by both two-color pulses.

We can define a transfer matrix for the combination of the pair of two-color pulses in the $|B\rangle$, $|D\rangle$ basis as:

$$U_{TC}^{BD} = \begin{pmatrix} e^{i\theta} & 0 \\ 0 & 1 \end{pmatrix} = e^{i\theta}|B\rangle\langle B| + |D\rangle\langle D|$$

If written in the $|0\rangle$, $|1\rangle$ basis instead, one can see that the interaction is really a rotation by an angle θ around a vector pointing in the equatorial plane set by the angle ϕ (neglecting any global phase). In the preparation exercises you will perform this rewriting and also compare the result with a NOT gate; a rotation around x, $R_x(\theta)$; and a rotation around y, $R_y(\theta)$.

Since any unitary operator can be written as;

$$U = e^{i\alpha}R_x(\beta)R_y(\gamma)R_x(\delta)$$

We can see that a combination of two-color pulses can describe any arbitrary unitary operator. Furthermore, all our two-color pulses use the robust sechyp expression shown in equation (1).

Quantum state tomography

To determine the quantum state of our qubit a straightforward absorption measurement will give the relative population of $|0\rangle$ and $|1\rangle$. However, in order to determine the phase of the state we need to perform a full quantum state tomography (QST).

A quantum state tomography involves three separate measurements. Since a quantum state collapses during readout this means that the full experiment including the preparation of the quantum state that should be read out need to be repeated three times. The goal of each measurement is to determine the projection on one of the three Bloch sphere axes x, y, or z.

As mentioned before a simple absorption measurement can determine the value of the z projection since it lies along what we call the population axis. To determine, e.g., the x projection, the state vector is rotated $-90°$ around the y axis so that the x axis now becomes vertical and lies along the population axis (the old z axis), i.e., the x-value of the Bloch vector will now be the new z-value. After this rotation if an absorption measurement is performed it will, just as before, give a value between -1 and $+1$, but now this value tells us about the x projection of our original state. Similarly, the y projection can be determined by first performing a rotation around x with $90°$ to bring the y axis value of the Bloch vector to the population axis before doing the absorption measurement.

These rotations of our state vectors can be performed using our pair of two-color pulses as described in the previous section, provided we choose the correct ϕ and θ, something you will determine in the preparation exercises.

Experimental setup

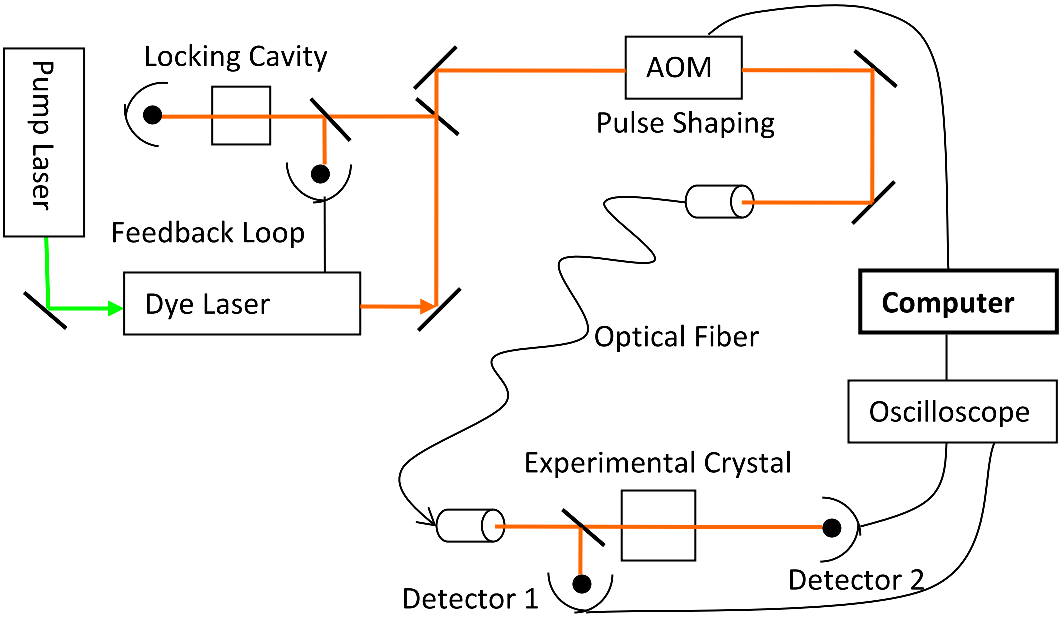

Figure 6) Experimental setup of the lab.

A schematic experimental setup for the lab can be seen in figure 6. The Praseodymium ions are excited by a wavelength of 606 nm. There are very few tunable sources in this wavelength region, and we are using a dye laser, which is pumped by a solid state Neodymium laser. Unfortunately also the best commercial dye lasers are too noisy and spectrally broad for controlling the Pr quantum states with good precision. In order to get the laser stable enough to match the coherence times of the Pr ions, we have therefore built a frequency stabilization system, which is indicated by the feedback loop in Figure 6. This system narrows the linewidth of the laser down to around 10 Hz, which can be considered rather amazing when remembering that 606 nm is equal to around 500 THz, meaning the laser is frequency stabilized down to 1 part in 10^{14}.

The light from the laser is continuous, and we are then employing Acousto-Optic Modulators (AOMs) to shape the light into having the desired amplitude envelopes and frequency chirps. The main element of the AOM is a piezo-electric ultrasound speaker attached to a crystal. A RF pulse signal is sent to the ultrasound speaker and the generated sound wave in the crystal creates a refractive index variation in the crystal, which acts like a grating for the incoming light. In the lab the AOMs are controlled from a computer, where a Matlab program is used to specify the pulse parameters.

After the pulse shaping, the light goes through a fiber (which also cleans up the spatial mode) and over to the experimental crystal with the Pr^{3+}ions. In order to avoid phonon excitations (which drastically shorten the coherence time) it is necessary to cool the crystal to around 4 K. This is done using a cryostat that operates with liquid Helium. There are two detectors in the experiment. One is situated just before the crystal, and used as a reference detector, and the other one just after the crystal, will detect the actual experimental signal. One can divide and subtract the reference signal from the experimental signal in order to reduce noise due to signal fluctuations (noise) in the input pulses. The detectors are then connected to an oscilloscope which in turn is connected to a computer where the signal can be analyzed.