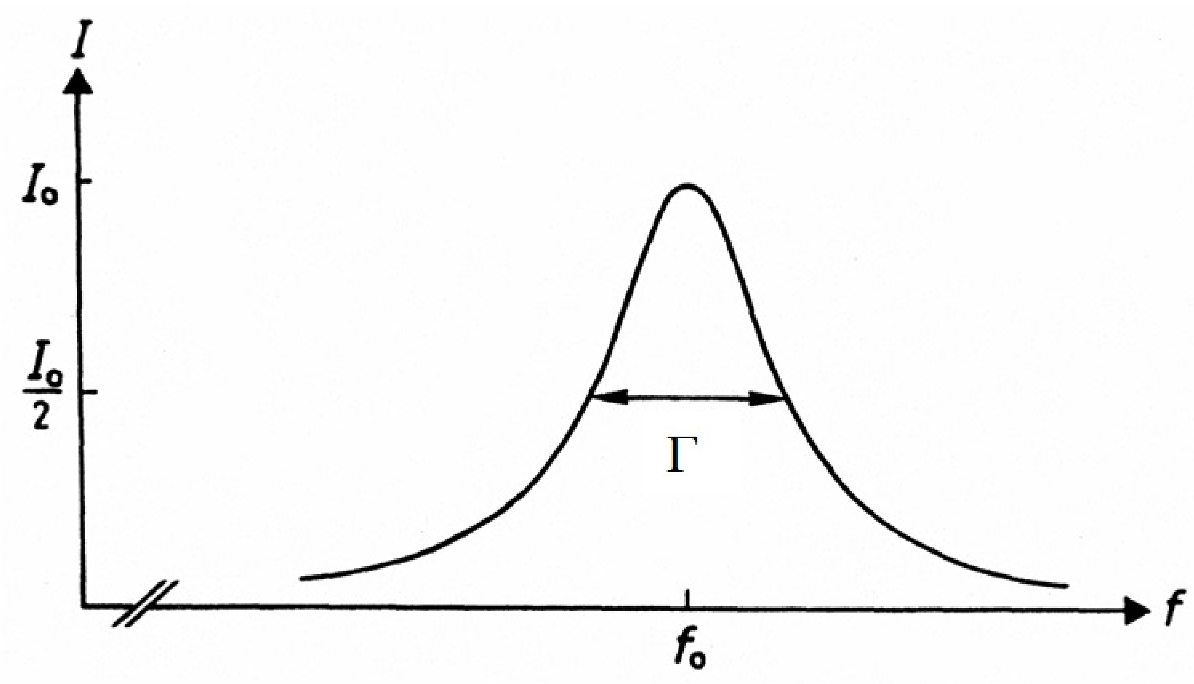
Line widths and profiles

Spectrophysics Ch. 8 (H2)

Source	Type	Profile
In the atom	Natural	Lorentz
In the light source	Collisional	Lorentz
	Power	Lorentz
	Self absorption	--
	Doppler	Gauss
In the detector	Instrumental	Airy
		N-slit
		Appox. Gauss

Line width. Lorentz profile



$$I(f) = I_0 \cdot \frac{\Gamma^2}{4(f - f_0)^2 + \Gamma^2}.$$

$$\Gamma = \Delta f_{\mathrm{FWHM}} = \frac{1}{2\pi\tau}$$

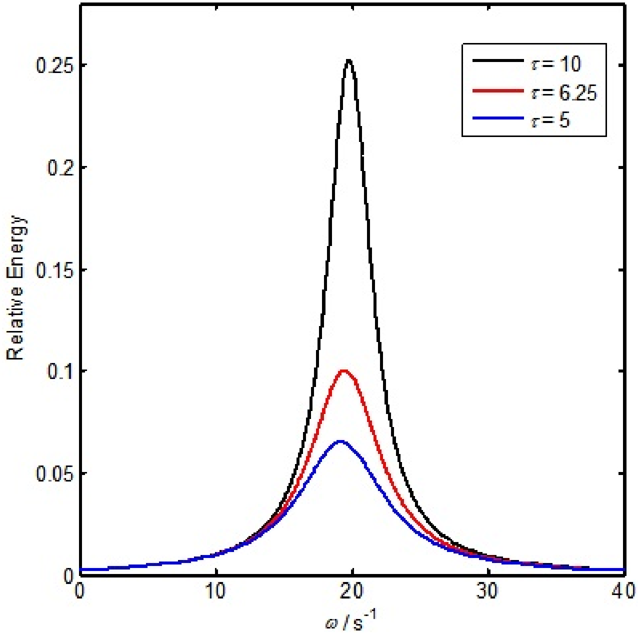
Damped and driven classical HO

$$\frac{d^2x}{dt^2} + \frac{1}{\tau}\cdot\frac{dx}{dt} + \omega_0^2 \cdot x = F\cos\omega t$$

$$x(t) = A\cdot\cos(\omega t - \delta)$$

$$I \sim A^2 = I(\omega_0)\cdot\frac{(\Delta\omega)^2}{(\omega_0-\omega)^2+(\Delta\omega)^2}, \quad \Delta\omega = 1/\tau$$

This is called a Lorentz profile



Note that, just like the Heisenberg result for a quantum system, $\Delta\omega_{\mathrm{FWHM}} = \frac{1}{\tau}$, i.e long time to measure gives a sharp resonance

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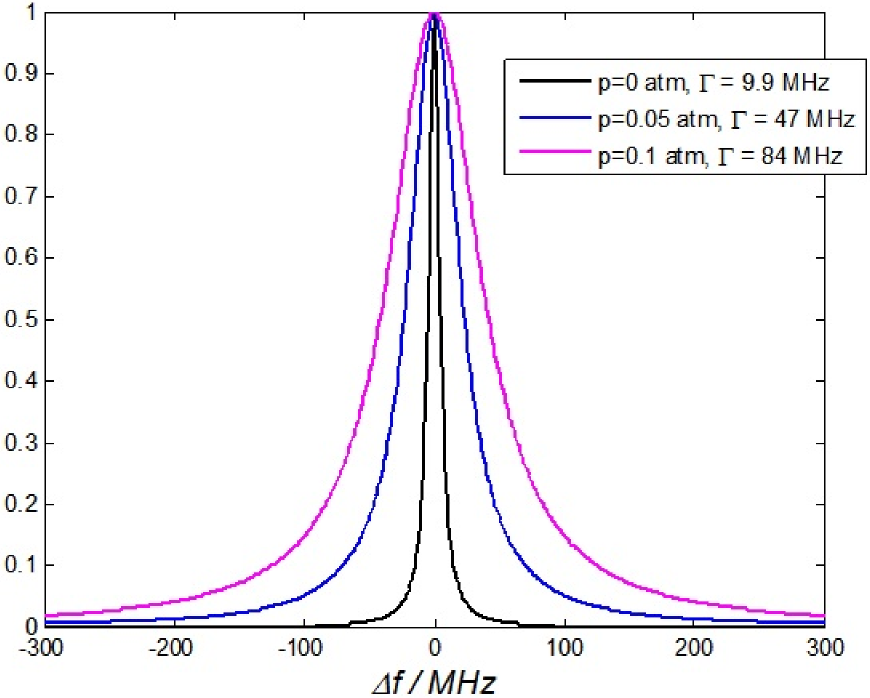
Pressure/collisional broadening in Na



The time between two collisions in a gas, *t*, is the *mean free path* divided by the *mean velocity*.

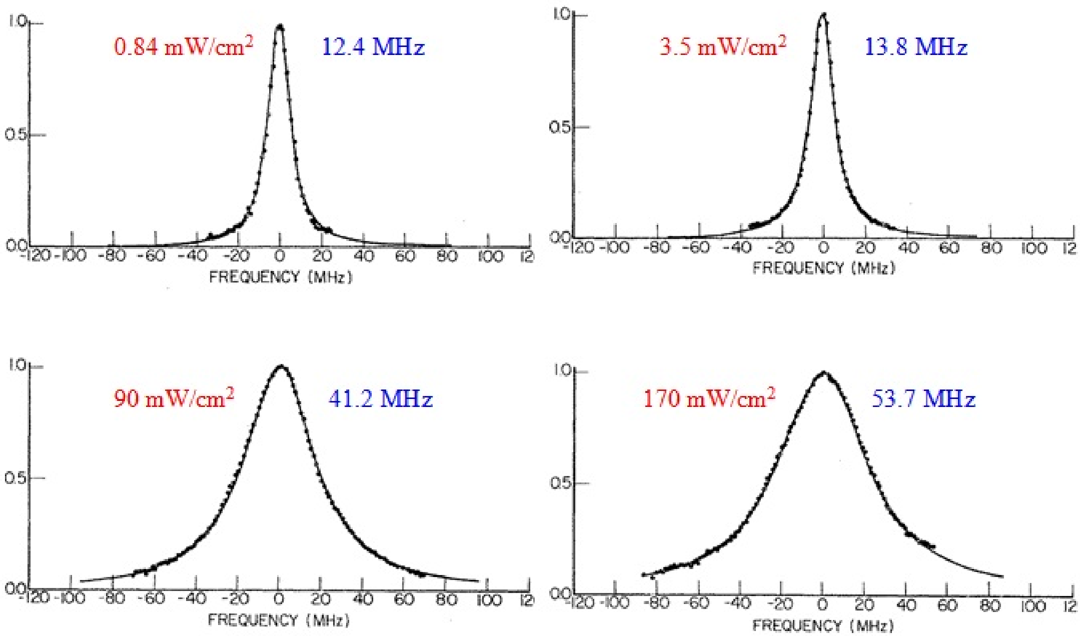
$$t = \frac{\sqrt{kTm}}{4\sqrt{2\pi}d^2 p} \sim \frac{\sqrt{T}}{p}$$, where p is the pressure.

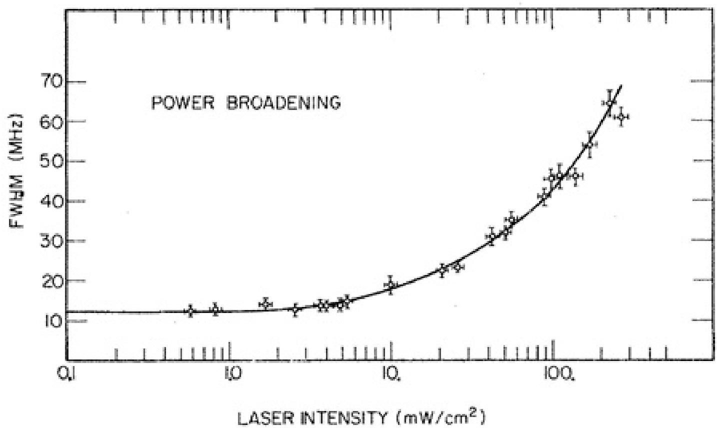
$$A_{\mathrm{col}} = \text{probability per second for a collision} \sim \frac{p}{\sqrt{T}}$$

$$\tau_i = \frac{1}{\sum_j A_{ij} + A_{\mathrm{col}}}. \quad A_{\mathrm{col}} \nearrow \Rightarrow \tau \searrow \Rightarrow \Delta f = \frac{1}{2\pi\tau} \nearrow$$

Power broadening in the 3s $^2S_{1/2}$ - 3p $^2P_{3/2}$ line in Na

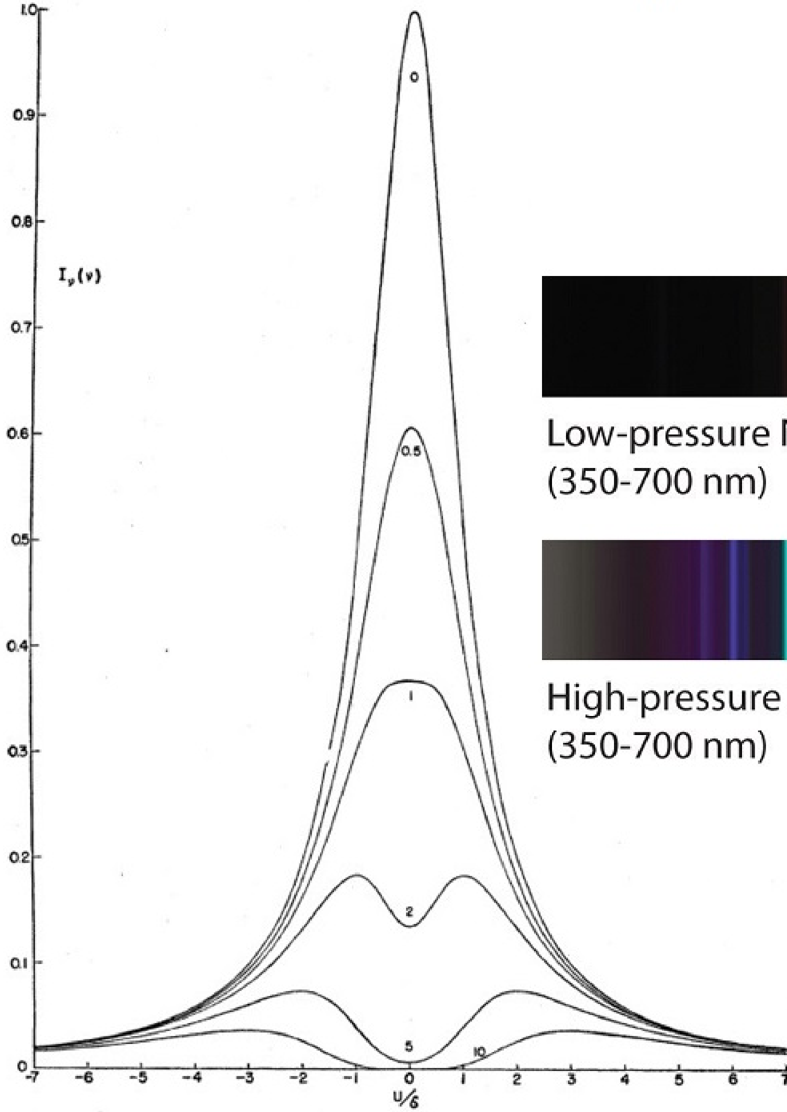
Line width (MHz) vs. laser intensity (mW/cm^2). Note the Lorentzian profile

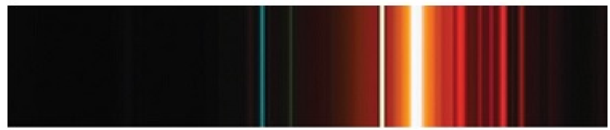




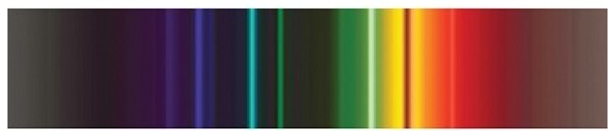
M.L.Citron *et al*, Phys. Rev. **A16**, 1507 (1977)

Self absorption





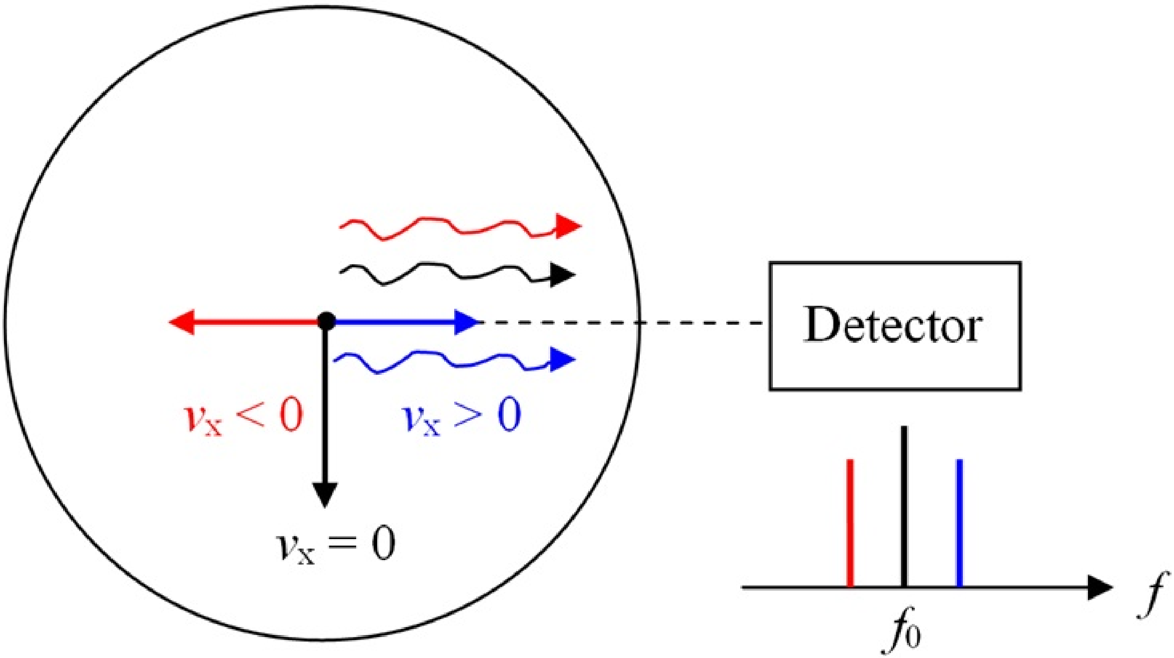
Low-pressure Na spectrum
(350-700 nm)



High-pressure Na spectrum
(350-700 nm)

Cowan, R.D. and Dieke, G.H., Rev. Mod. Phys. **20**, 418 (1948)

Doppler broadening.



$$\text{If } v_x << c \text{ then } f = f_0 \cdot (1 \pm \frac{v_x}{c})$$

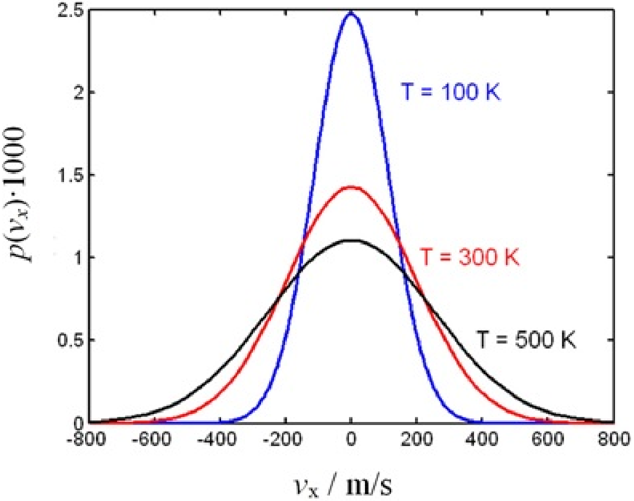
Doppler broadening due to Maxwell's velocity distribution in 1-dimension

$p(v_x)$ is the probability to find an atom with a velocity of v_x

$$p(v_x) = \sqrt{\frac{m}{2\pi kT}} \cdot e^{-\frac{mv_x^2}{2kT}}.$$

Δv_x is "Full Width at Half Maximum"

$$\Delta v_x = 2 \cdot \sqrt{\frac{2kT}{m} \cdot \ln(2)} \qquad \Delta v_x \nearrow \text{ when } T \nearrow$$



$$f = f_0 \cdot (1 \pm \frac{v_x}{c}) \Rightarrow \Delta f = \frac{f_0}{c} \Delta v_x = \frac{f_0}{c} 2\sqrt{2\ln 2} \cdot \sqrt{\frac{kT}{m}} = (\frac{k}{m} = \frac{R}{M})$$

$$\frac{f_0}{c} 2\sqrt{2\ln 2} \cdot \sqrt{R} \cdot \sqrt{\frac{T}{M}}.$$

$$\boxed{\Delta f = a \cdot f_0 \cdot \sqrt{\frac{T}{M}}. \quad a = 7.16 \cdot 10^{-7} \ (\text{g/mol})^{1/2} \text{K}^{-1/2}.}$$

Note M should be given in the unit 1 g / mol.
For example, for Na $M = 23$.

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	Doppler	**Gauss**
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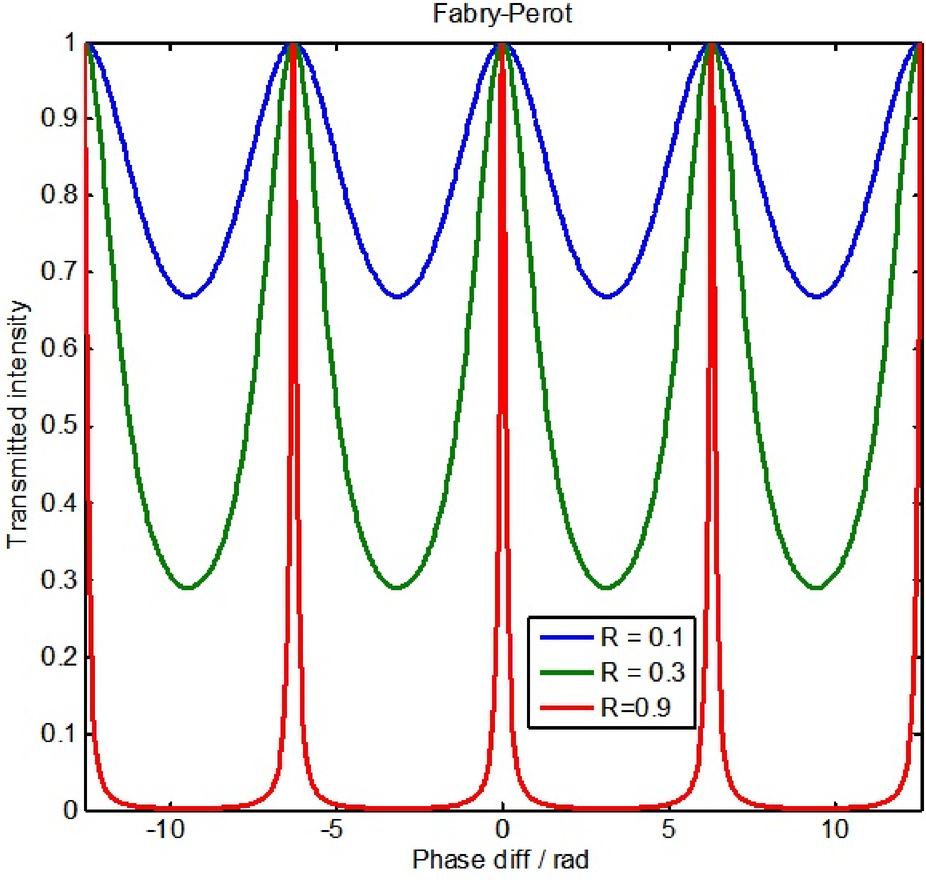
		***N*-slit**
		Appox. Gauss

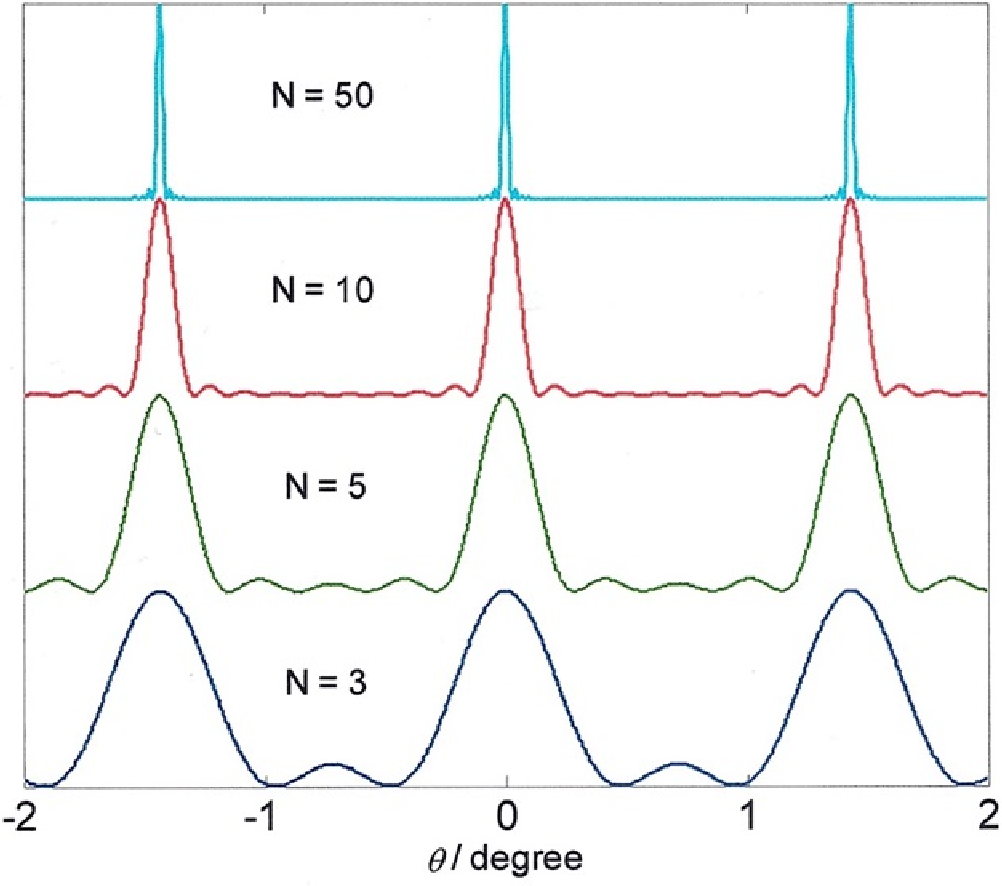
The Airy function



$$I = I_0 \cdot \frac{1}{1 + \frac{4R}{(1-R)^2}\sin^2(\delta/2)}, \quad \Delta\delta_{\text{FWHM}} = 2 \cdot \frac{1-R}{\sqrt{R}}.$$

Thus as R increases the line width decreases

Grating ($N \approx 10^5$)



$$I = I(0) \cdot \left(\frac{\sin \beta}{\beta} \right)^2 \cdot \left(\frac{\sin N\alpha}{N \sin \alpha} \right)^2$$

$$\beta = \frac{1}{2} ka \sin \theta \text{ and } \alpha = \frac{1}{2} kd \sin \theta$$

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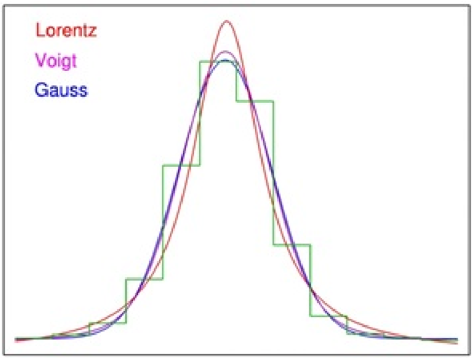


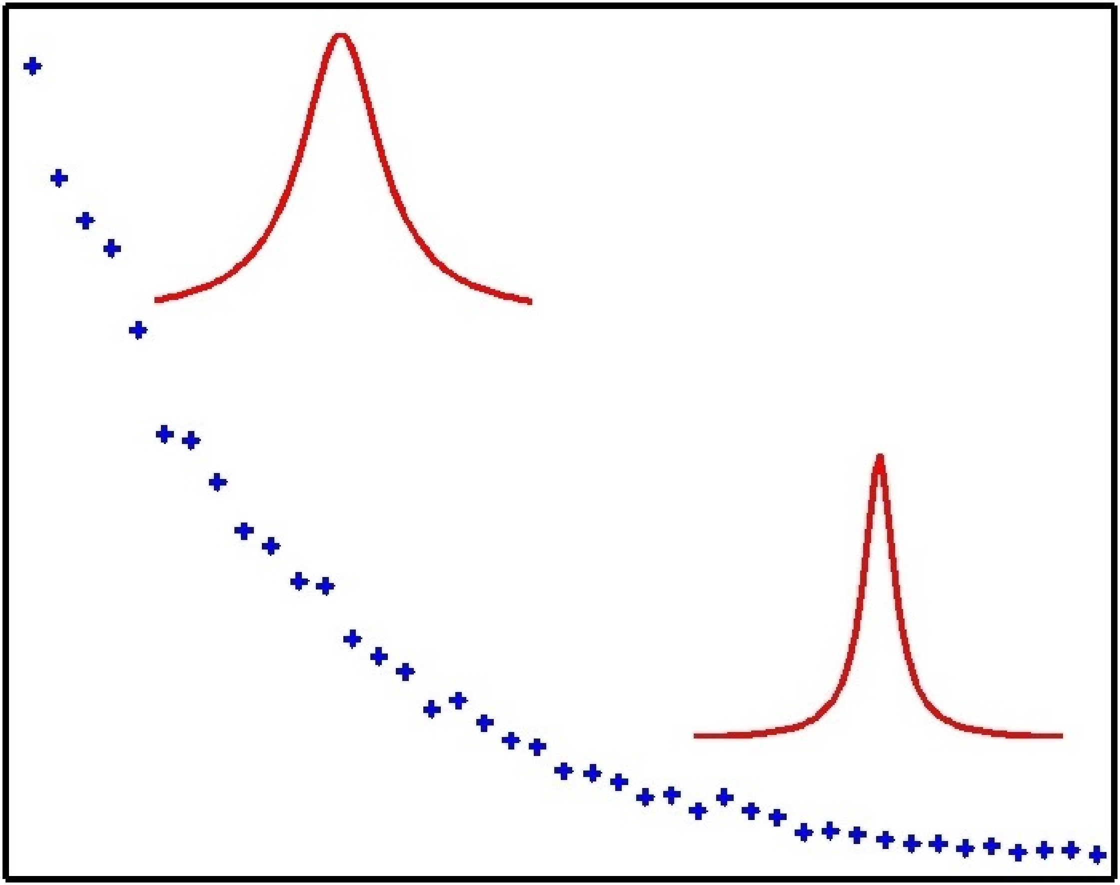
Voight = convolution of Lorentz and Gauss

Na-example
(3s - 3p 5890 Å, lifetime 16 ns)

Broadening	Δf / GHz	$\Delta\lambda$ / Å
Natural	0.01	0.0001
Power, $I = 170$ mW/cm^2	0.054	0.0006
Pressure, $p = 0.1$ atm	0.084	0.0009
Doppler (600 K)	1.86	0.022
Instrument. Fabry Perot: $d = 1$ cm, $R = 0.9$	0.25	0.003
Instrument. Grating 1200 ℓ/mm, 50 µm slit	30	0.35

Below the natural line width using “old light”

$$N(t) = N(0) \cdot e^{-t/\tau}$$



Time