Magnetic dipole interactions

Magnetic interaction - internal

Spin-orbit interaction: $\hat{H}_{SO} = -\hat{\mu}_S \cdot \hat{B}_L$ ✔

Hyperfine interaction $\hat{H}_{\text{hfs}} = -\hat{\mu}_I \cdot \hat{B}_J$

Magnetic interaction - external

Zeeman and Paschen-Back effect (Foot 5.5, SP 3.9)

We will study three magnetic effects in an atom.
The energy is always given by:

$E = < -\hat{\mu} \cdot \hat{B} >_{\Psi_0}$,

and we need only determine how the magnetic moment ($\hat{\mu}$) and the magnetic field ($\hat{B}$) should be calculated and what wavefunctions (Ψ_0) to use in the calculation in each case.

$$\hat{\mu}_L = -\frac{e}{2m}\hat{L}$$

$$\hat{\mu}_S = -g_s \cdot \frac{e}{2m} \cdot \hat{S}, \quad g_s = 2$$

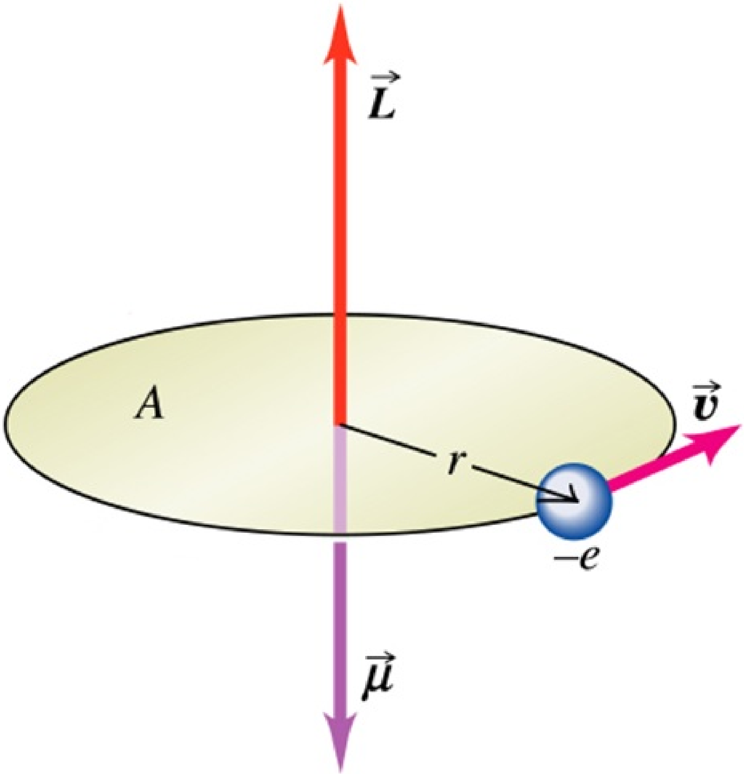
$$\hat{\mu}_{\text{tot}} = -\frac{e}{2m} \cdot (\hat{L} + 2\hat{S})$$

Units

We: $L \sim \hbar,\ \hat{\mu} = -\frac{e}{2m}\hat{L}$

Foot: $L \sim 1,\ \hat{\mu} = -\mu_B \hat{L},\ \mu_B$ (Bohr magneton) $= \frac{e\hbar}{2m}$

Magnetic dipole moment due to the orbital motion of an electron



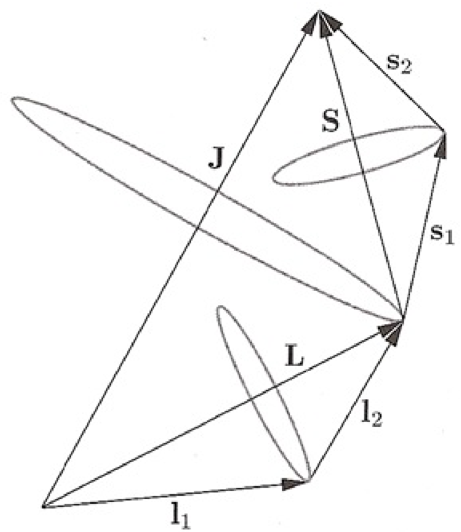
$$\overline{\mu}_\ell = -\frac{e}{2m}\overline{\ell}$$

$$\overline{L} = \sum \overline{\ell}_i \Rightarrow \overline{\mu}_L = -\frac{e}{2m}\sum \overline{\ell}_i = -\frac{e}{2m}\overline{L}$$

Angular momentum couplings of two electrons in open subshells

Energy structure	Angular momentum coupling	Wavefunctions	Eigenfunctions to
Configuration	-	$\left\|\ell_i, m_{\ell_i}\right\rangle \cdot \left\|s_i, m_{s_i}\right\rangle$	$\hat{\ell}_i^2,\ \hat{\ell}_{iz},\ \hat{s}_i^2,\ \hat{s}_{iz}$
Term ^{2S+1}L	$\hat{L} = \hat{\ell}_1 + \hat{\ell}_2,\ \hat{S} = \hat{s}_1 + \hat{s}_2$	$\left\|L,\ M_L,\ S,\ M_S\right\rangle$	$\hat{L}^2,\ \hat{L}_z,\ \hat{S}^2,\ \hat{S}_z$
Level $^{2S+1}L_J$	$\hat{J} = \hat{L} + \hat{S}$	$\left\|L,\ S,\ J,\ M_J\right\rangle$	$\hat{L}^2,\ \hat{S}^2,\ \hat{J}^2, \hat{J}_z$

$$\left|\ell_i, m_{\ell_i}\right\rangle \cdot \left|s_i, m_{s_i}\right\rangle = R_{n_i \ell_i}(r_i) \cdot Y_{\ell_i m_{\ell_i}}(\theta_i, \varphi_i) \cdot \chi_{m_{s_i}}(sz_i)$$



An explicit, LS-coupled, non antisymetrized wave function for a 2 electron configuration, $n_1\ell_1 n_2\ell_2$

$$\left|LM_L SM_S\right\rangle = \sum_{m_{\ell_1} m_{\ell_2}} \sum_{m_{s_1} m_{s_2}}$$

$$C(\ell_1 m_{\ell_1} \ell_2 m_{\ell_2} : LM_L) \cdot C(s_1 m_{s_1} s_2 m_{s_2} : SM_S)$$

$$R_{n_1\ell_1}(r_1) Y_{\ell_1 m_{\ell_1}}(\theta_1, \varphi_1) \chi_{s_1 m_{s_1}}(s_{z_1}) \cdot R_{n_2\ell_2}(r_2) Y_{\ell_2 m_{\ell_2}}(\theta_2, \varphi_2) \chi_{s_2 m_{s_2}}(s_{z_2})$$

Numerical

An explicit, LSJ-coupled, non antisymetrized wave function for a 2 electron configuration, $n_1\ell_1 n_2\ell_2$

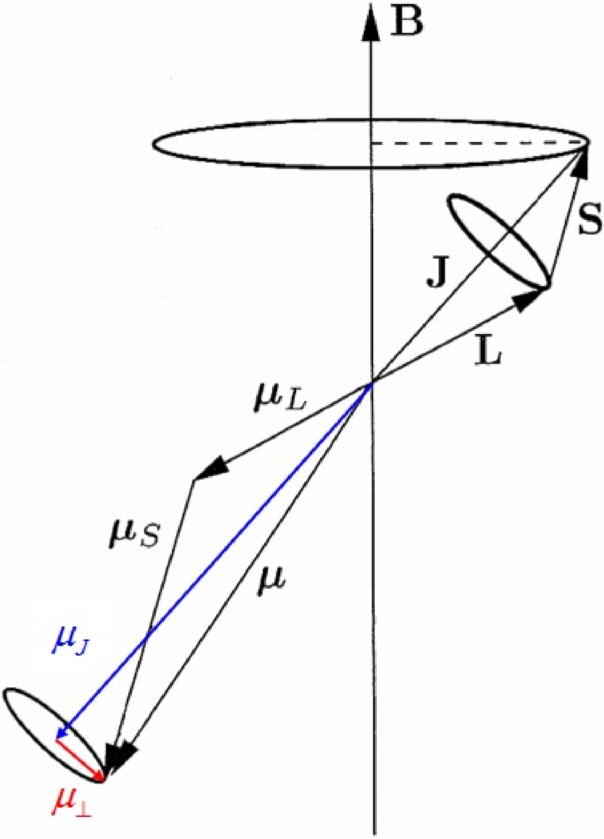
$$\left|LSJM\right\rangle = \sum_{M_L M_S} \sum_{m_{\ell_1} m_{\ell_2}} \sum_{m_{s_1} m_{s_2}}$$

$$C(LM_L SM_S : JM)\cdot C(\ell_1 m_{\ell_1} \ell_2 m_{\ell_2} : LM_L)\cdot C(s_1 m_{s_1} s_2 m_{s_2} : SM_S)$$

$$R_{n_1\ell_1}(r_1) Y_{\ell_1 m_{\ell_1}}(\theta_1,\varphi_1)\chi_{s_1 m_{s_1}}(s_{z_1})\cdot R_{n_2\ell_2}(r_2) Y_{\ell_2 m_{\ell_2}}(\theta_2,\varphi_2)\chi_{s_2 m_{s_2}}(s_{z_2})$$

Numerical

Zeeman effect in the vector model

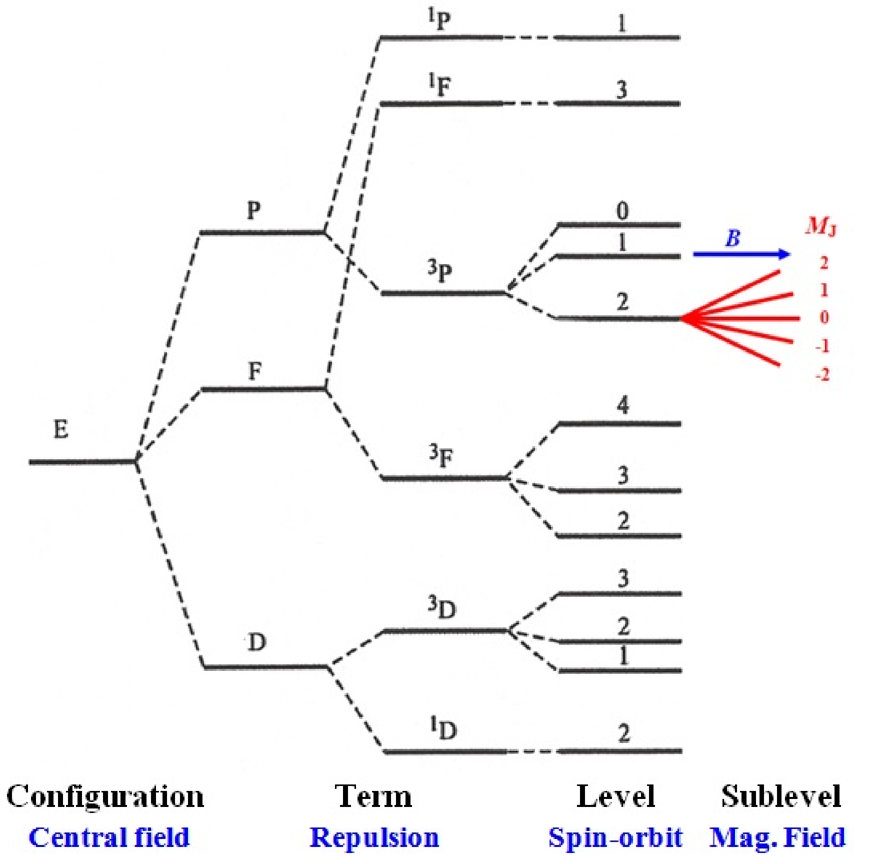


$$\bar{\mu} = \mu_J \cdot \bar{e}_J + \mu_\perp \cdot \bar{e}_\perp$$

Weak field =>

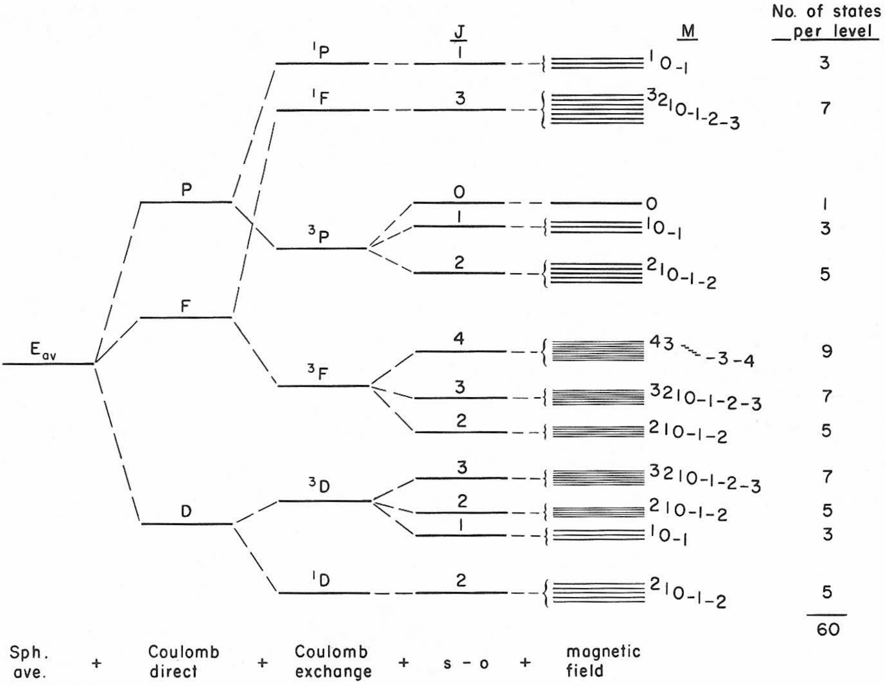
- **L** and **S** precess much faster around **J** than **J** around **B**
- The net effect of $\mu_\perp$ is zero

pd-configuration LSJ-coupling and a magnetic field



Numerical example for 2p3d in O V, energies in cm^{-1}

E(2p3d) =	701810	Kinetic and central part of electrostatic
ΔE (P - D) =	8980	Direct part of electrostatic repulsion
ΔE (^1F - ^3F) =	15074	Exchange part of electrostatic repulsion
ΔE ($^3F_4$- $^3F_3$) =	235	Spin-orbit magnetic energy
ΔE_{mag} (2 - 1) =	0,7	Magnetic energy separation in a 1T field


No. of states per level
J
M
E_av
P
F
D
¹P
¹F
³P
³F
³D
¹D
1
3
0
1
2
4
3
2
3
2
1
2
1 0 -1
3 2 1 0 -1 -2 -3
0
1 0 -1
2 1 0 -1 -2
4 3 -3 -4
3 2 1 0 -1 -2 -3
2 1 0 -1 -2
3 2 1 0 -1 -2 -3
2 1 0 -1 -2
1 0 -1
2 1 0 -1 -2
3
7
1
3
5
9
7
5
7
5
3
5
60
Sph. ave.
+
Coulomb direct
+
Coulomb exchange
+
s - o
+
magnetic field

Selection rules E1 (electric dipole) transitions

$\Delta J = 0, \pm 1$ not 0 to 0

Only one electron can change orbital, i.e. $n\ell$

$\Delta \ell = \pm 1$

$\Delta M_J = 0, \pm 1$ not 0 to 0 if $\Delta J = 0$

If perfect LS-coupling, i.e real states = basis states

$\Delta S = 0$

$\Delta L = 0, \pm 1$ not 0 till 0

Preparatory exercises

1. The ground configuration in neutral Cd is $5s^2$ and the first excited configuration is 5s5p. In the experiment you will, among other lines, see the transition 5s5p ^3P - 5s6s ^3S.
   a) Give the *LS* notation for the possible transitions between these terms.
   b) You will find a green ($\lambda = 508.582$ nm), a turquoise ($\lambda = 479.992$ nm) and a blue ($\lambda = 467.816$ nm) line. Which of the transitions above correspond to the different colors?

2. **This exercise is essential to the lab and a solution must be presented before you are allowed to continue.**

   In the experiment you will study the transitions 5s5p $^1P_1$ - 5s5d $^1D_2$, 5s5p $^3P_2$ - 5s6s $^3S_1$, 5s5p $^3P_1$ - 5s6s $^3S_1$ and 5s5p $^3P_0$ - 5s6s $^3S_1$ in a weak magnetic field.
   a) Derive the Landé *g* factor assuming *LS* coupling for the levels involved in the four transitions.
   b) Draw large and nice diagrams showing the different Zeeman components that each of the 4 lines (not levels) split into in the magnetic field in the manner of Figure 3.16 in Spectrophysics or Figure 5.13 in Atomic Physics. Thus, choose a relative energy scale, with zero at the energy of the transition without magnetic field, and show the splittings in units of $\mu_B B$ along the x – axis. Let all Zeeman components have the same intensity.
   c) What is the state of polarization of each of the components?
   d) Which components do you expect to see in a direction parallel to the magnetic field?

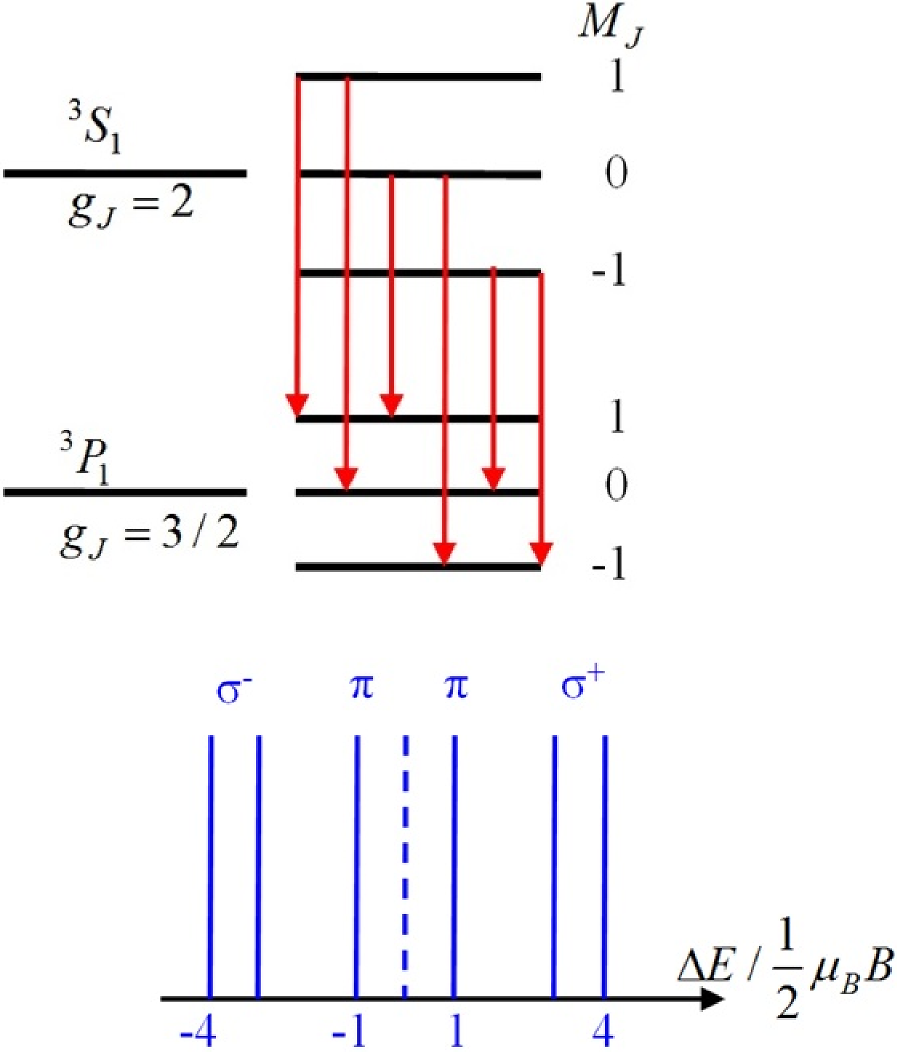
3. Let B = 0.5 T. How large is the smallest splitting between the components derived above?
   a) Expressed in eV
   b) Expressed in cm^{-1}
   c) Expressed in nm

4. Use Appendix 1 to answer the following. A Fabry-Perot interferometer operating in air have mirror surfaces with a reflectance of $R = 0.85$ and separated by 3.085 mm. We use a light source with a wavelength of 500 nm.
   a) What is the free spectral range expressed in cm^{-1} and in nm.
   b) What is the line width expressed in cm^{-1} and nm.
   c) Does the size of the rings increase or decrease in higher spectral orders?

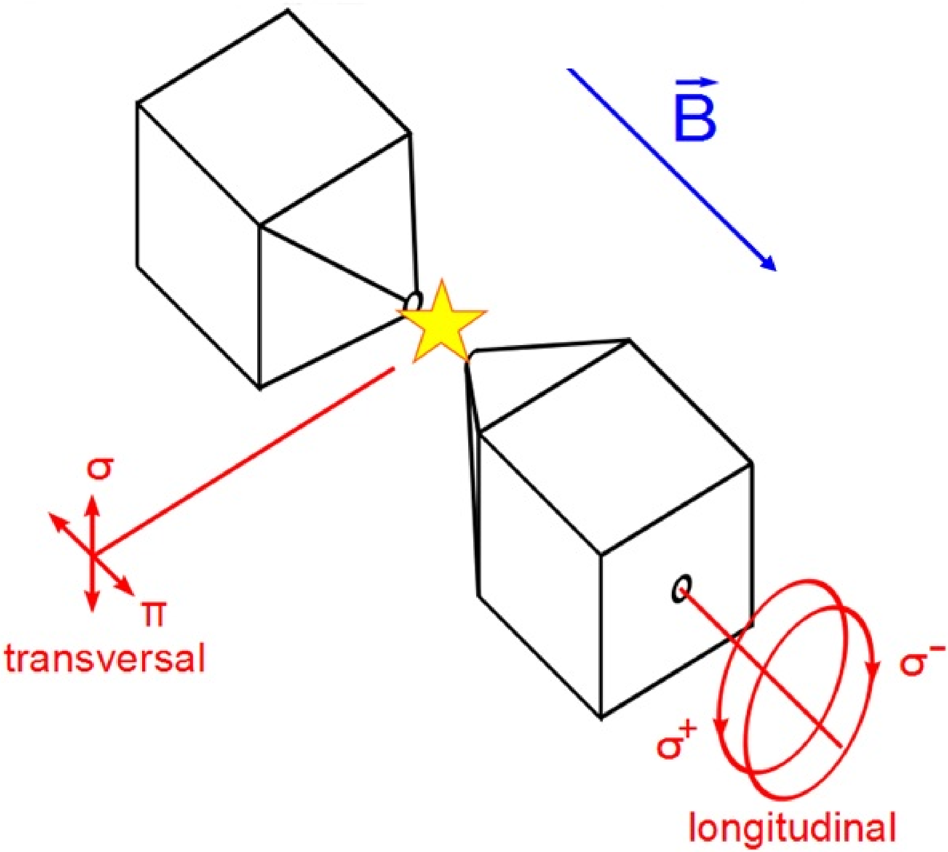
5. Use Appendix 2 to answer the following. What is the polarization of light when the electric field is described by the expressions below?

   a) $\overline{E} = E_0 \cdot (\overline{e}_x \cdot \sin(kz - \omega t) + \overline{e}_y \cdot \cos(kz - \omega t))$

   b) $\overline{E} = 5 \cdot \overline{e}_x \cdot \sin(kz - \omega t + \pi / 2) + 3 \cdot \overline{e}_y \cdot \sin(kz - \omega t)$

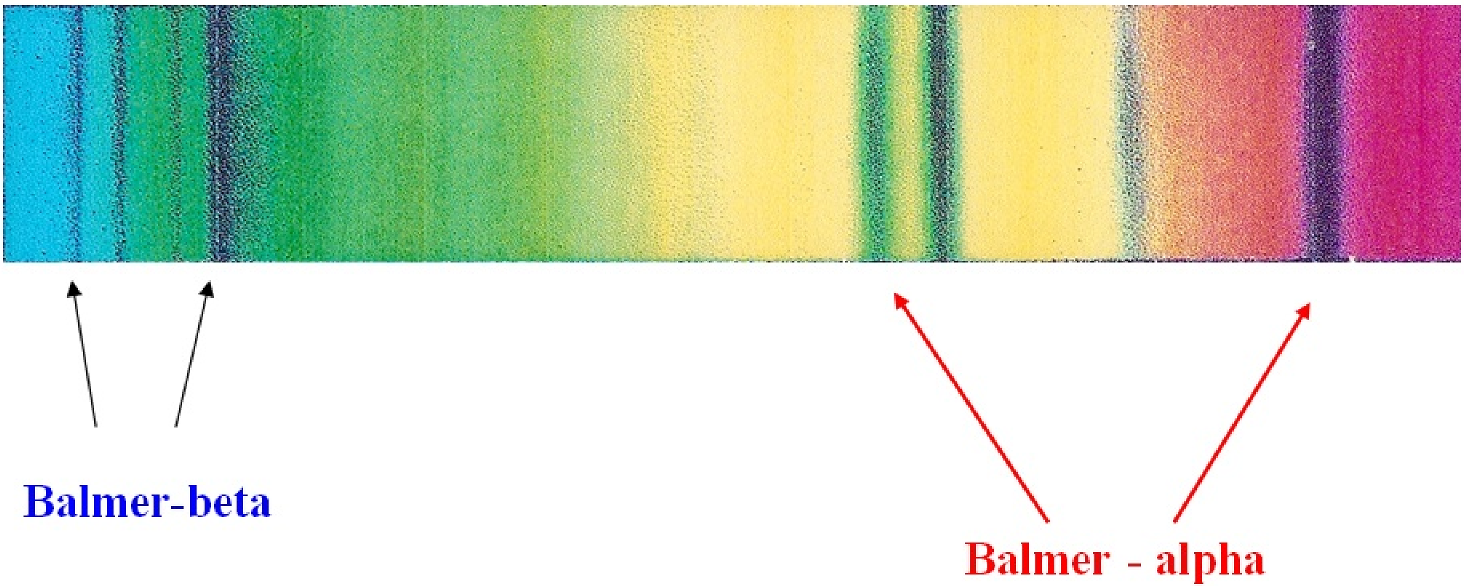

M_J
1
0
-1
$^3S_1$
$g_J = 2$
1
0
-1
$^3P_1$
$g_J = 3/2$
σ⁻
π
π
σ⁺
$\Delta E / \frac{1}{2}\mu_B B$
-4
-1
1
4

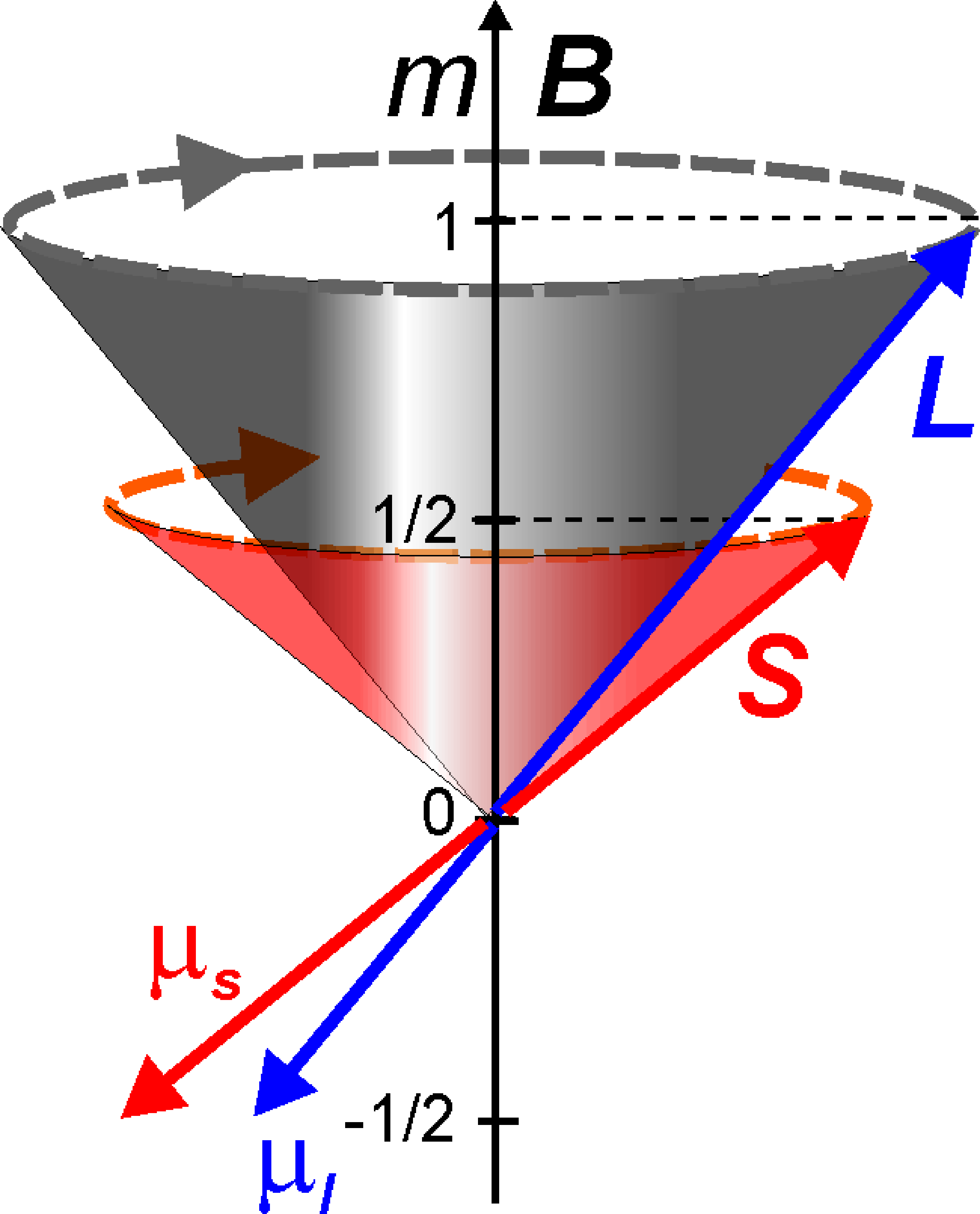
Zeeman effect and the polarization of light



Viewed in absorption: $\Delta M_J = \pm 1 \Rightarrow \sigma^{\pm}, \; \Delta M_J = 0 \Rightarrow \pi$

Magnetic effects on the spectrum from a white dwarf. Magnetic field about 6000 T!! (Paschen-Back effect)





m
B
1
L
1/2
S
0
μs
-1/2
μl

Zeeman vs. Paschen-Back in a ^{3}P

Zeeman	Paschen-Back
$E = E(^3P) +$	$E = E(^3P) +$
$\frac{1}{2}\beta_{LS} \cdot [J(J+1) - L(L+1) - S(S+1)] +$	$\mu_B \cdot B \cdot (M_L + 2M_S) +$
$\mu_B \cdot B \cdot g_J \cdot M_J$	$\beta_{LS} \cdot M_L \cdot M_S$
$\mu_B B = \frac{1}{10}\beta_{LS}$	$\mu_B B = 10 \cdot \beta_{LS}$