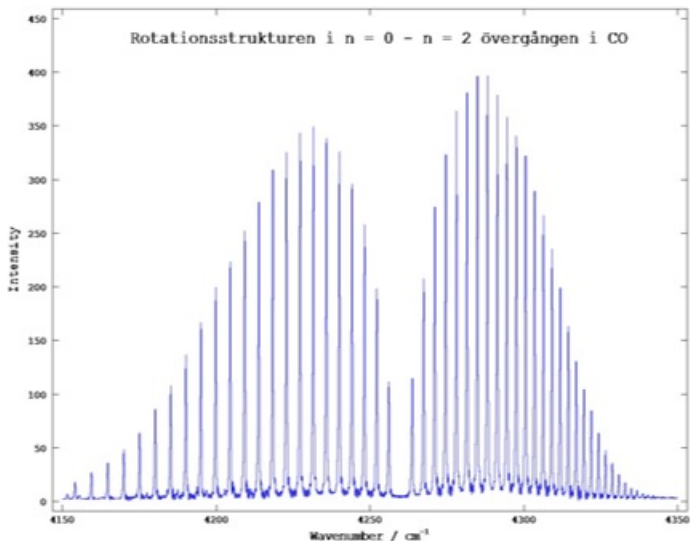# Structure and Spectra of Diatomic Molecules

Spectrophysics Ch 5 and 6. Exercises: 41, 42 and H43.
Focus on the vibration and rotation, that is unique to molecules.
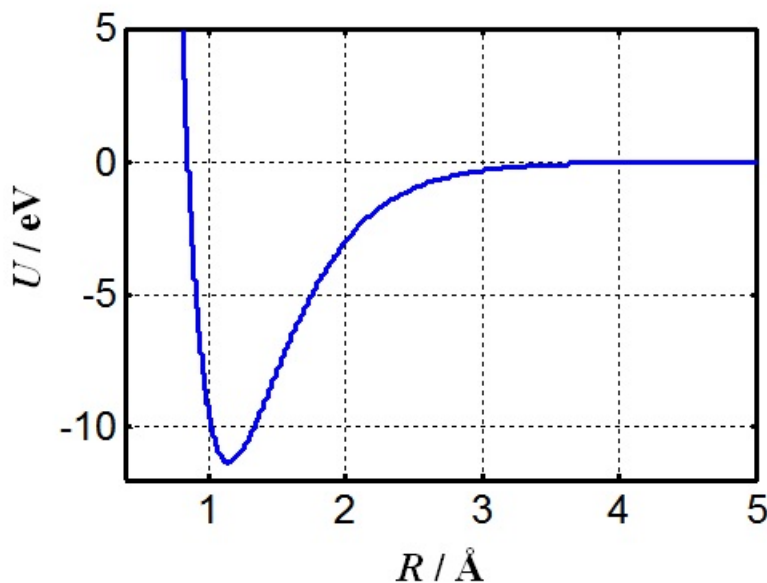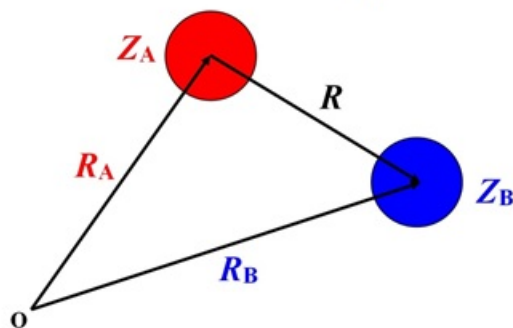
- **Energy = electronic + vibrational + rotational**

- Bonding and anti-bonding electronic states
- Photodissociation (Ozon)

- Vibration: $E_{vib} = \hbar\omega_0(v + \frac{1}{2})$

- Rotation: $E_{rot} = \dfrac{\hbar^2}{2\mu r^2} \cdot \ell(\ell+1)$

- Roto-vibrational transitions:
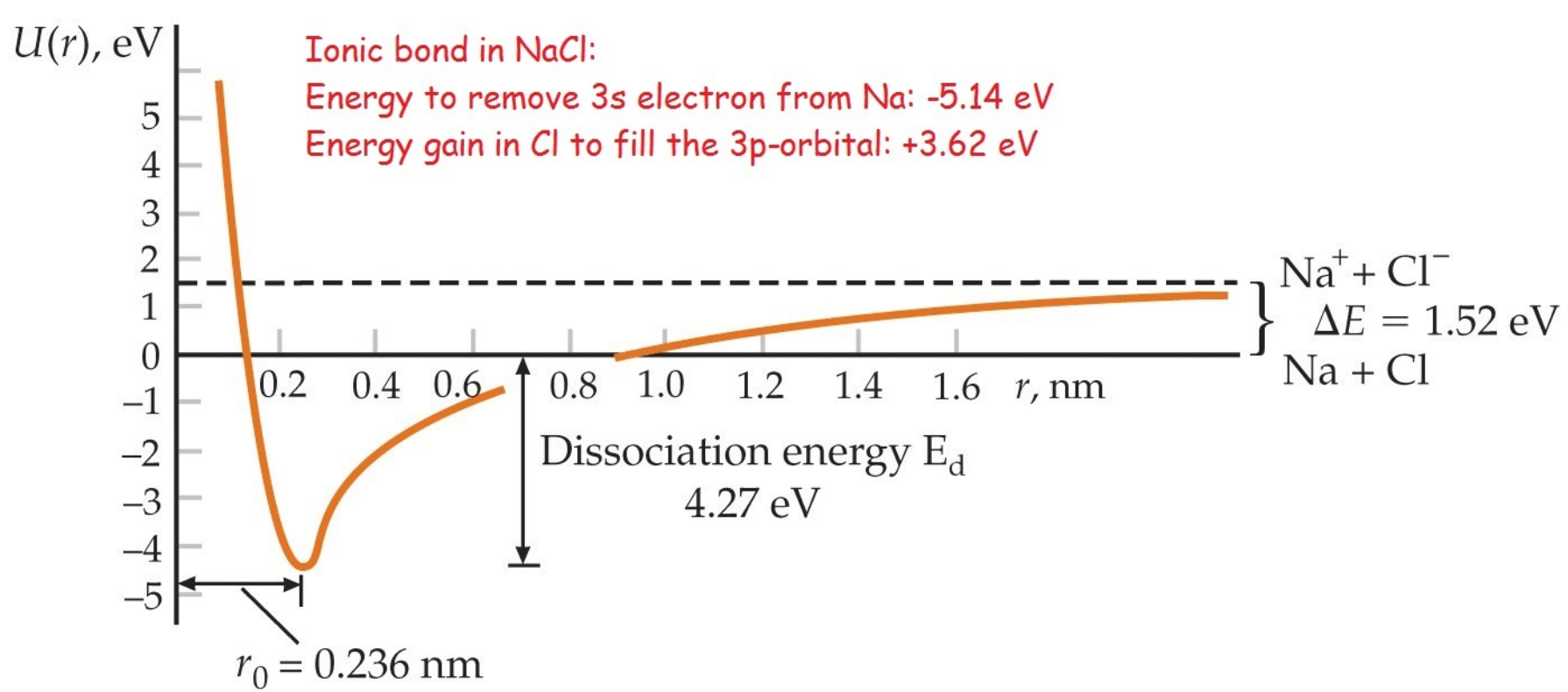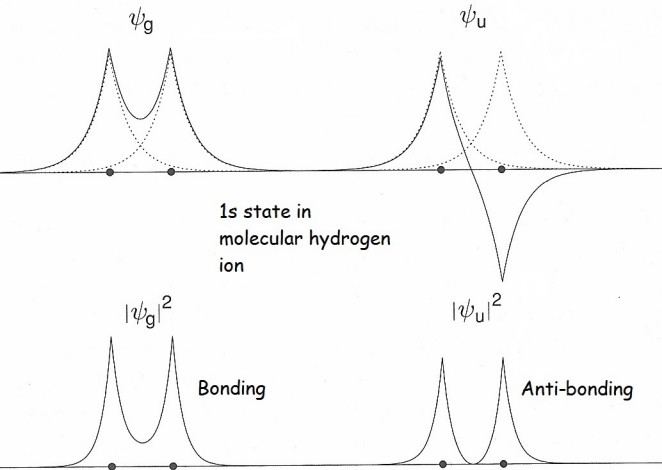

Rotationsstrukturen i n = 0 – n = 2 övergången i CO
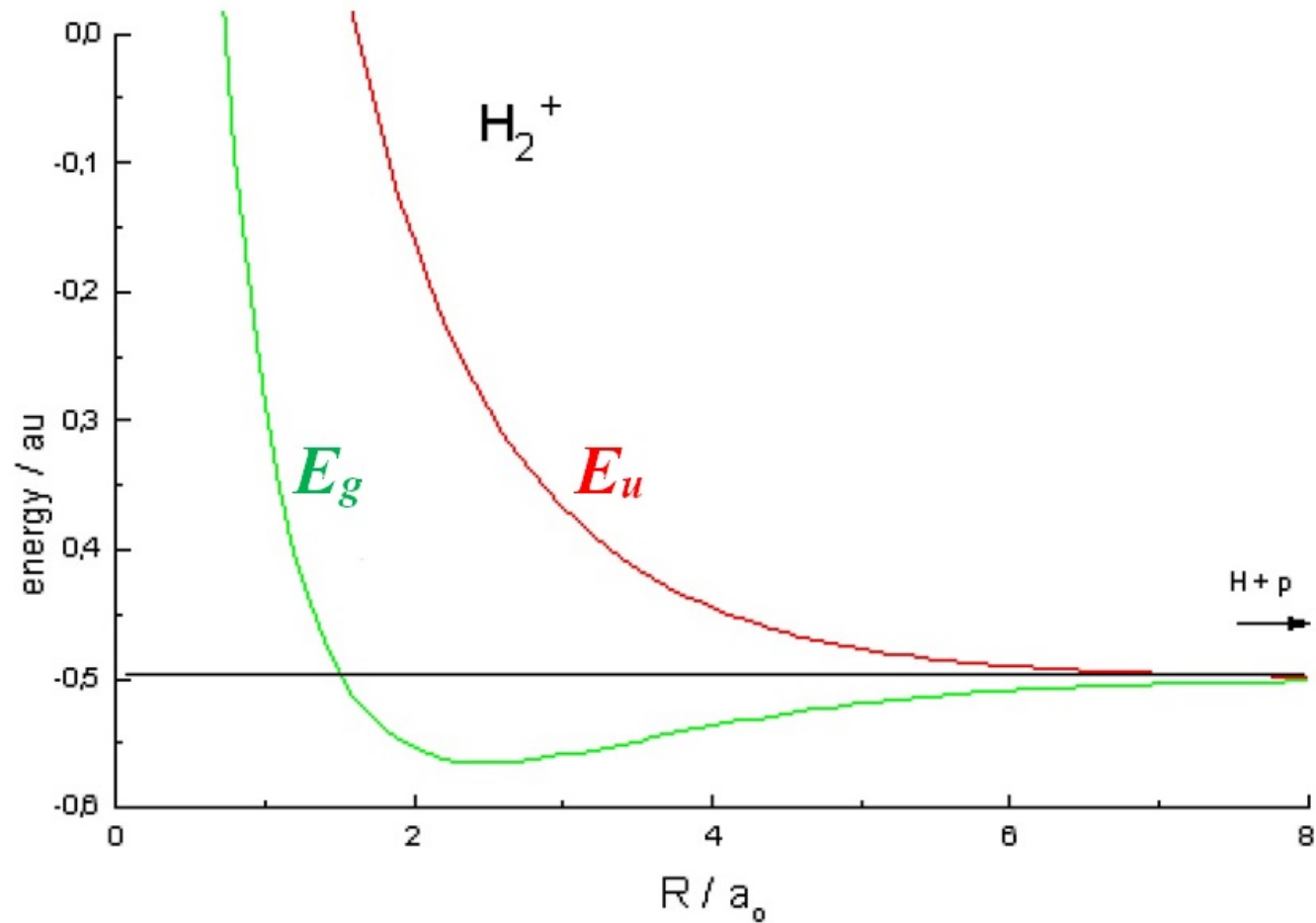
- Basic physics regarding the green house effect

Potential in a diatomic molecule with N electrons

$$V(\bar{R},\bar{r}_1,\bar{r}_2,.....,\bar{r}_N) = -\sum_{i=1}^{N}\frac{Z_A e^2}{4\pi\varepsilon_0\left|\bar{r}_i-\bar{R}_A\right|} - \sum_{i=1}^{N}\frac{Z_B e^2}{4\pi\varepsilon_0\left|\bar{r}_i-\bar{R}_B\right|}$$

$$+ \sum_{\substack{i,j=1\\i>j}}^{N}\frac{e^2}{4\pi\varepsilon_0\left|\bar{r}_i-\bar{r}_j\right|} + \frac{Z_A Z_B e^2}{4\pi\varepsilon_0 R}$$

$U(r)$, eV

Ionic bond in NaCl:
Energy to remove 3s electron from Na: -5.14 eV
Energy gain in Cl to fill the 3p-orbital: +3.62 eV

$Na^+ + Cl^-$

$\Delta E = 1.52$ eV

Na + Cl

Dissociation energy E_d
4.27 eV

r, nm

$r_0 = 0.236$ nm

ψ_g ψ_u

1s state in
molecular hydrogen
ion

$|\psi_g|^2$ $|\psi_u|^2$

Bonding Anti-bonding

Potential curves in OH

E / eV

OH

$^4\Pi$

$^2\Sigma^-$

$a^4\Sigma^-$

$A\,^2\Sigma^+$ O(^{1}D) + H(^{2}S)

O(^{3}P) + H(^{2}S)

X $^2\Pi$

7.5

5

2.5

0

1   2   3   4   r(Å)

160-240 nm

$$3O_2 \rightarrow 2O + 2O_2 \rightarrow 2O_3$$

240-360 nm

$$2O_3 \rightarrow 2O + 2O_2 \rightarrow 3O_2$$
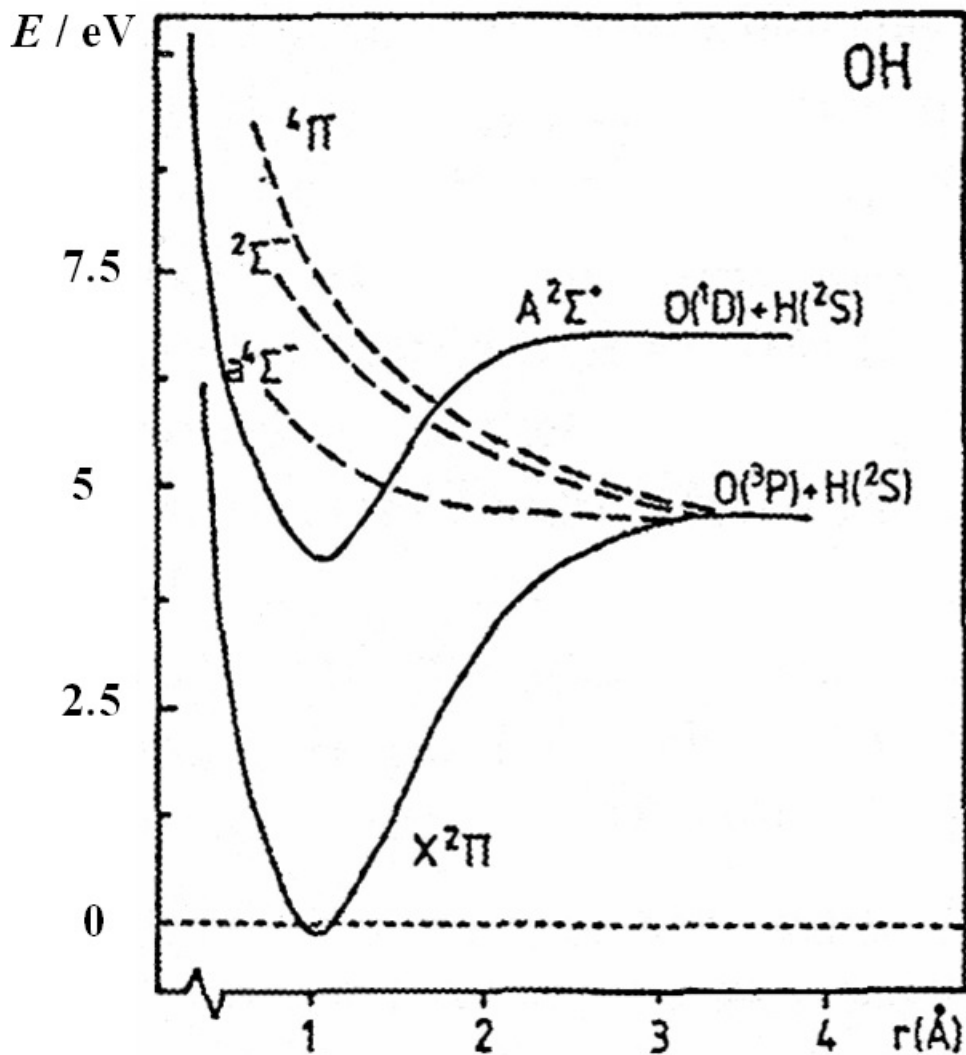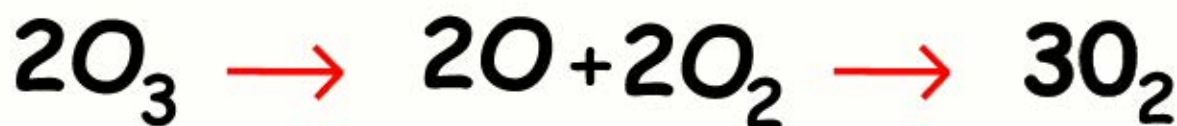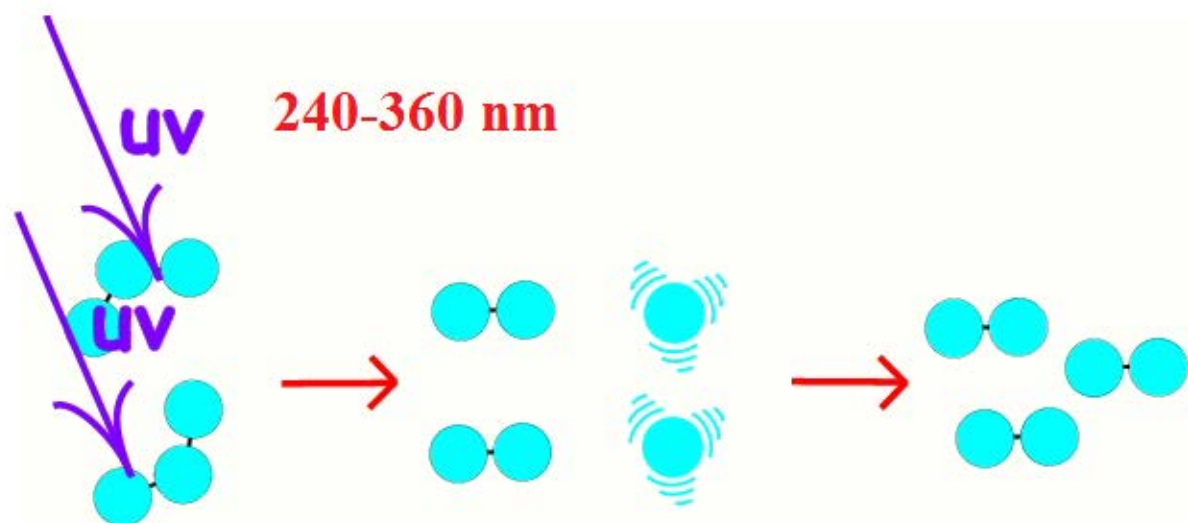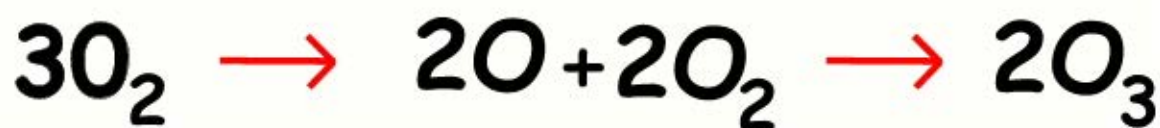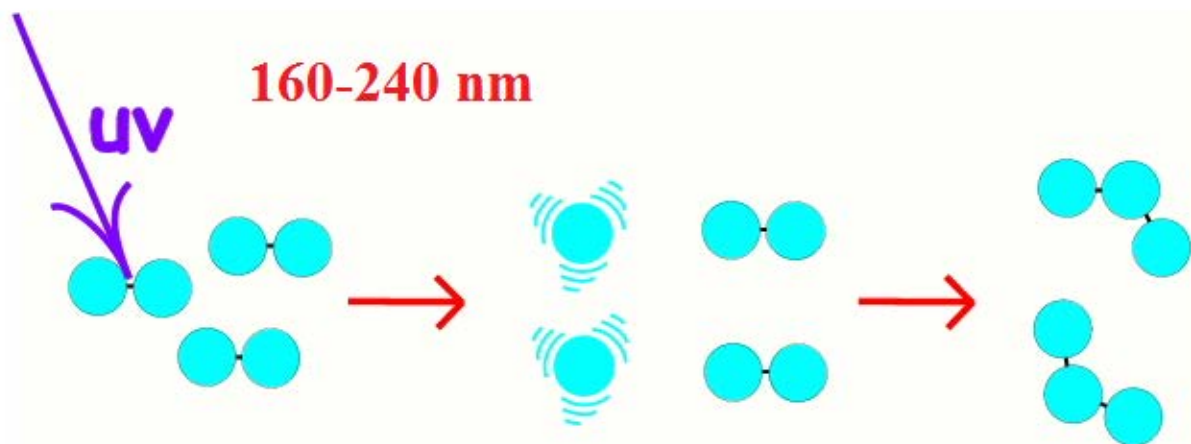
Absorption of Sunlight in the Atmosphere
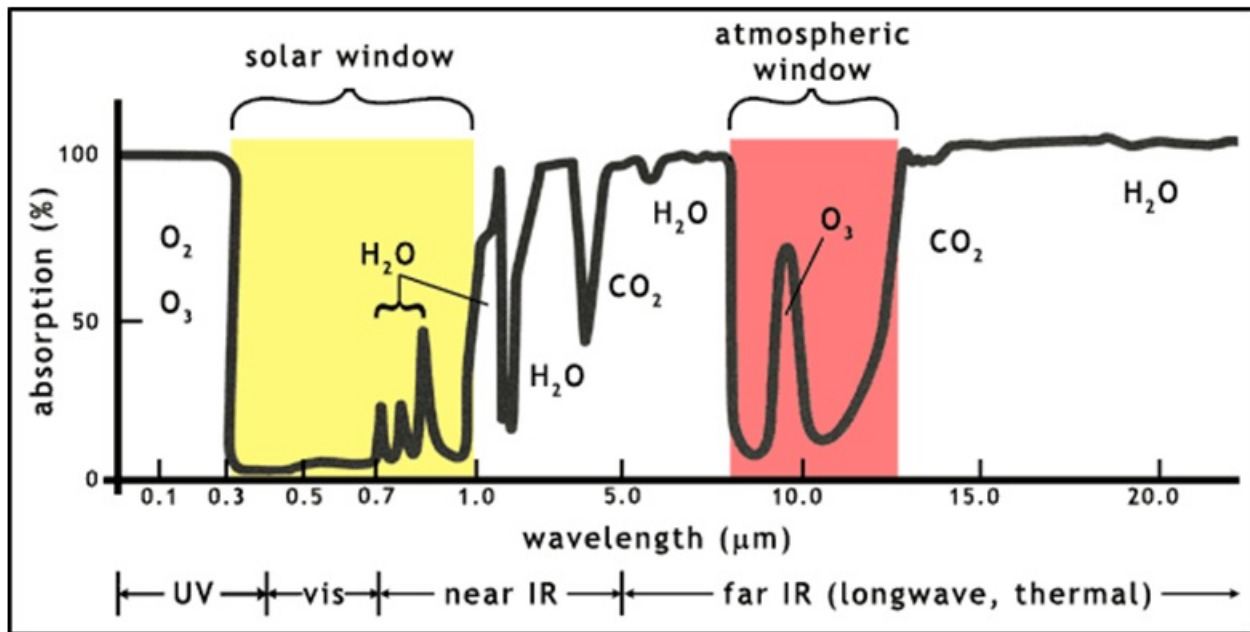
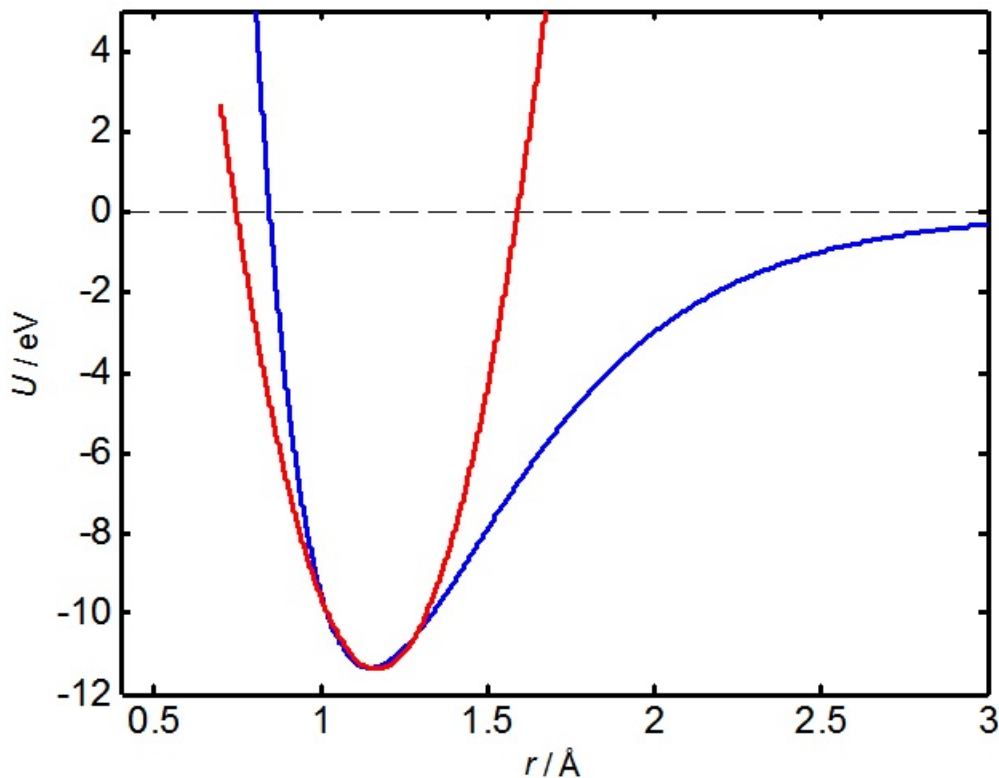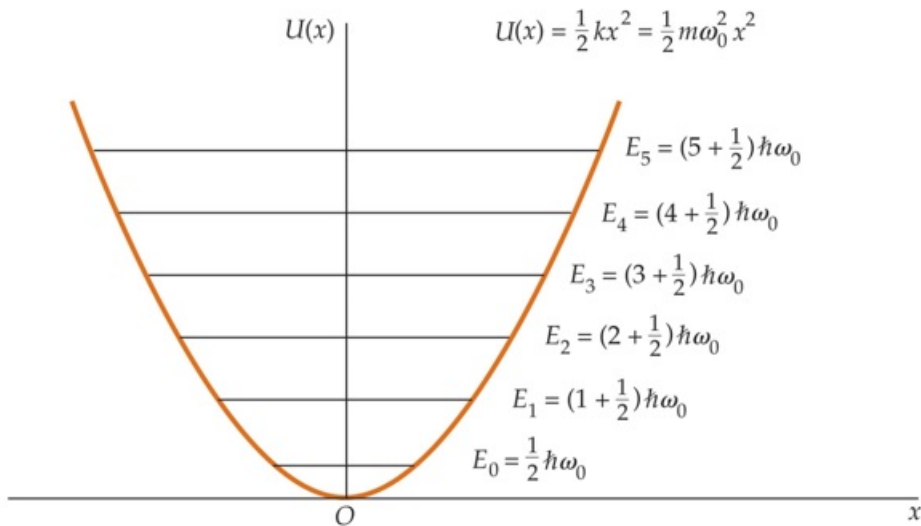| Atoms	Molecules
Energy <u>levels</u>	Energy <u>curves</u> (function of R)
Infinite number, converging to ionization limit	Limited number
$E \sim$ eV, $\lambda \sim$ UV, Visible	$E \sim$ eV, $\lambda \sim$ UV, Visible
Ionized up to Z	1, 2 times
-	Photo dissociation (ozone)
-	Vibration
-	Rotation

Vibrational Energy in a Diatomic Molecule

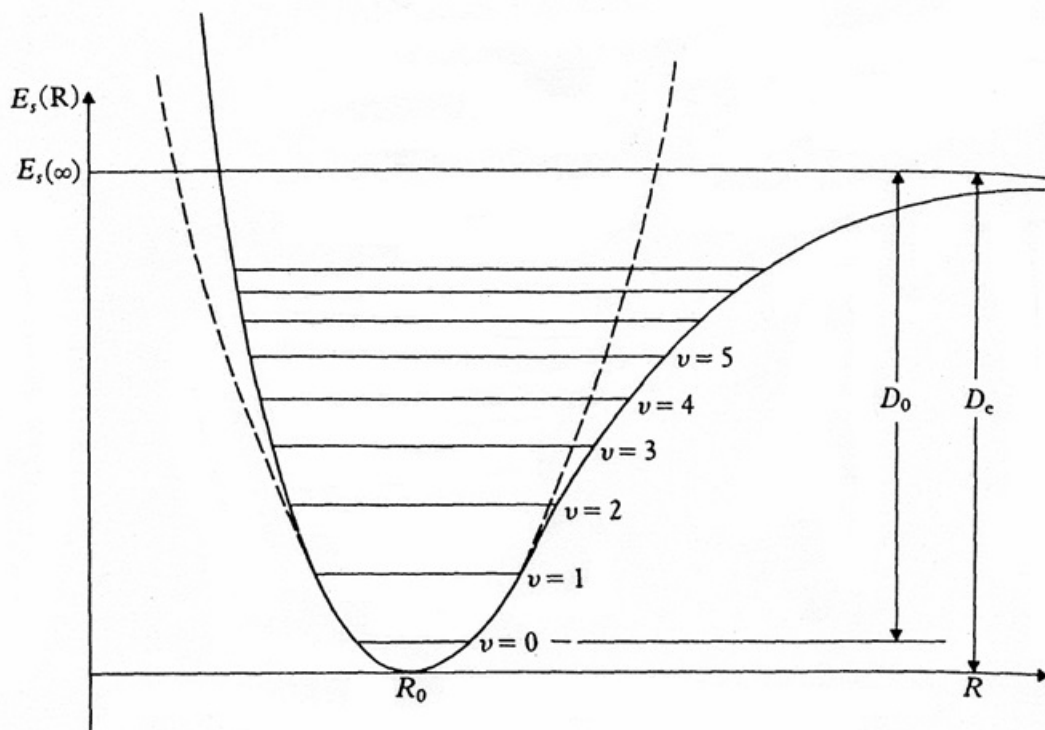### HO approximation (red) of the exact potential (blue) in CO

Quantized harmonic oscillator

$U(x)$

$U(x) = \frac{1}{2}kx^2 = \frac{1}{2}m\omega_0^2 x^2$

$E_5 = (5 + \frac{1}{2})\hbar\omega_0$

$E_4 = (4 + \frac{1}{2})\hbar\omega_0$

$E_3 = (3 + \frac{1}{2})\hbar\omega_0$

$E_2 = (2 + \frac{1}{2})\hbar\omega_0$

$E_1 = (1 + \frac{1}{2})\hbar\omega_0$

$E_0 = \frac{1}{2}\hbar\omega_0$

O

x

$$E = (\nu + \frac{1}{2}) \cdot \hbar\omega_0, \quad \nu = 0, 1, 2....$$
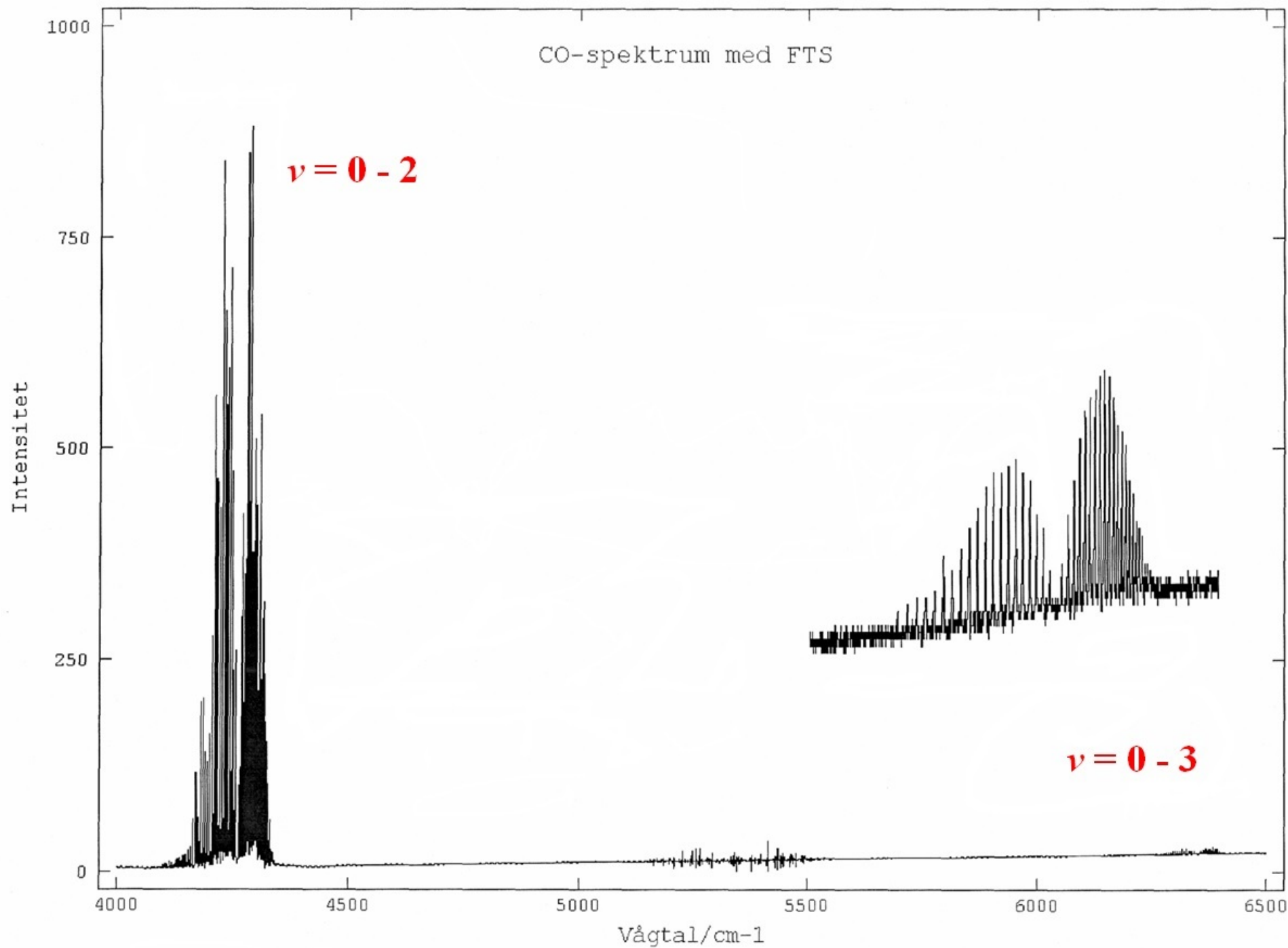
$$\Delta E = \hbar\omega_0$$

Comparison of vibrational energies in the HO approximation and in the real potential
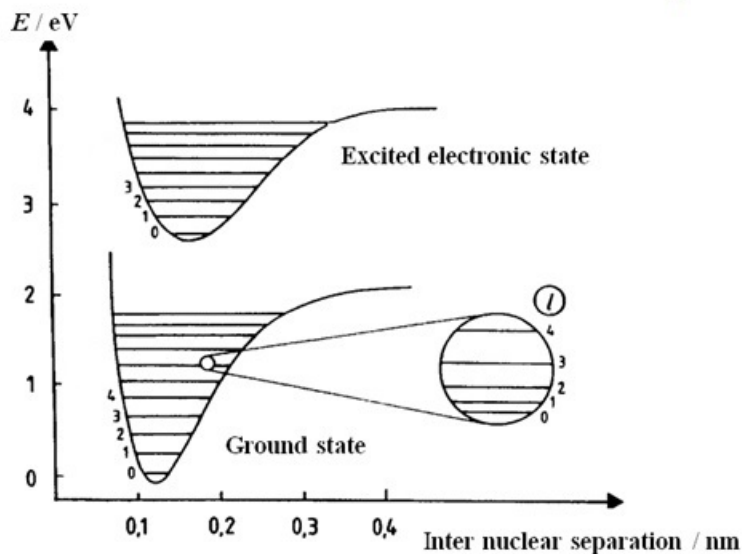

Selection rules:

HO: $\Delta v = 1$

Real: $\Delta v = 1, 2, 3$ (>1 weak)

CO-spektrum med FTS

$v = 0 - 2$

$v = 0 - 3$

Intensitet

Vågtal/cm-1

Energies in a Diatomic Molecule – Summary.

$$E = E_{elektron} + E_{vib} + E_{rot} = E_{elektron} + \hbar\omega_0(v + \frac{1}{2}) + \frac{\hbar^2}{2\mu r^2} \cdot \ell(\ell+1)$$

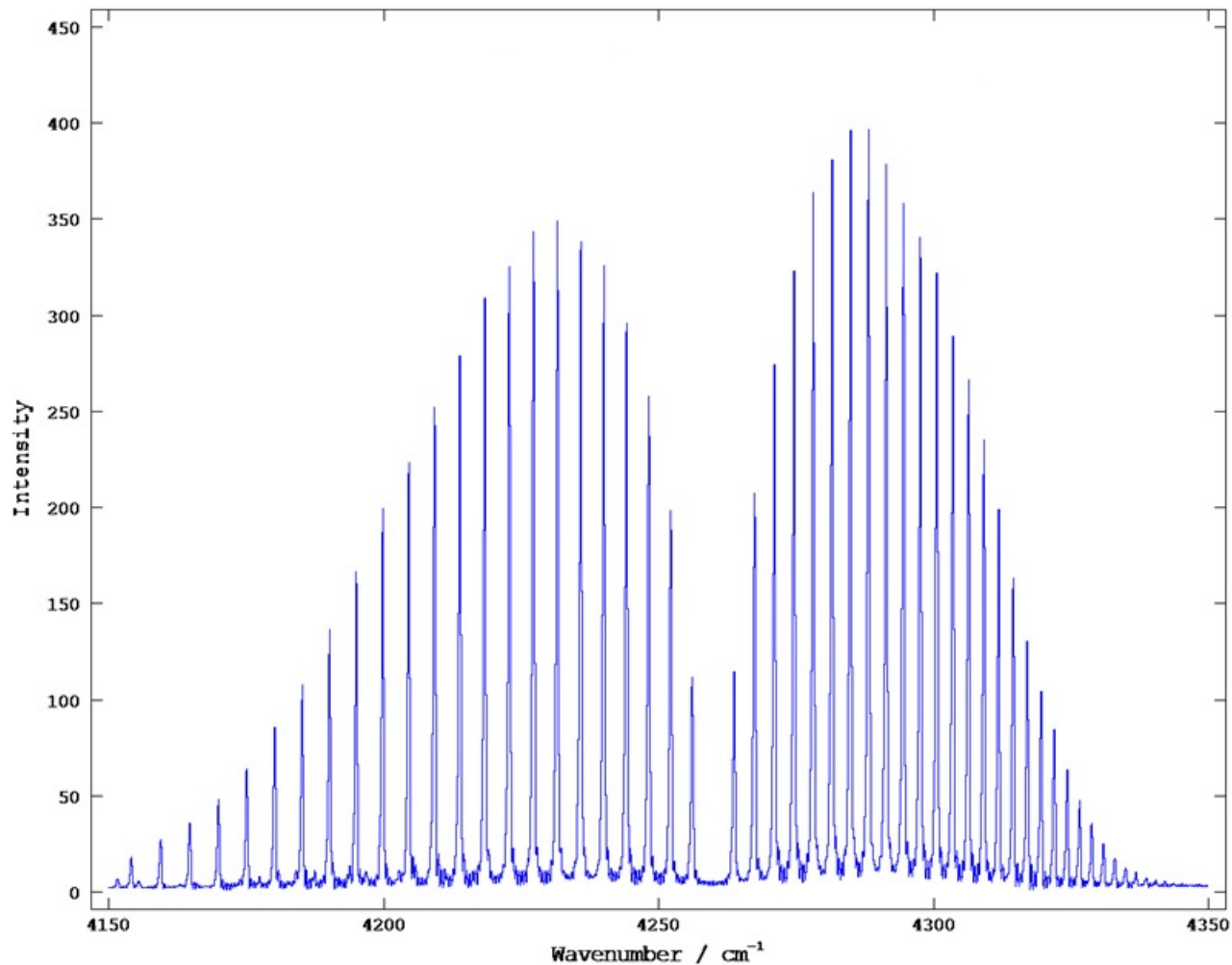## Orders of magnitude

	Electron	Vibration	Rotation
E / cm^{-1}	40000	1500	10
E / eV	5	0.2	0.001
λ / μm	0.3	6	1000
T / K	40000	1500	10

(Note the coincidence that energies expressed in cm^{-1} and as a temperature in K has almost the same numerical value, i.e. $1/(100 \cdot h \cdot c) = 5.035 \cdot 10^{22} \approx 2/(3 \cdot k) = 4.83 \cdot 10^{22}$!)

Rotational Fine Structure in the 0 − 2 Vibrational Transition in CO

Rotational fine structure in a vibrational transition.
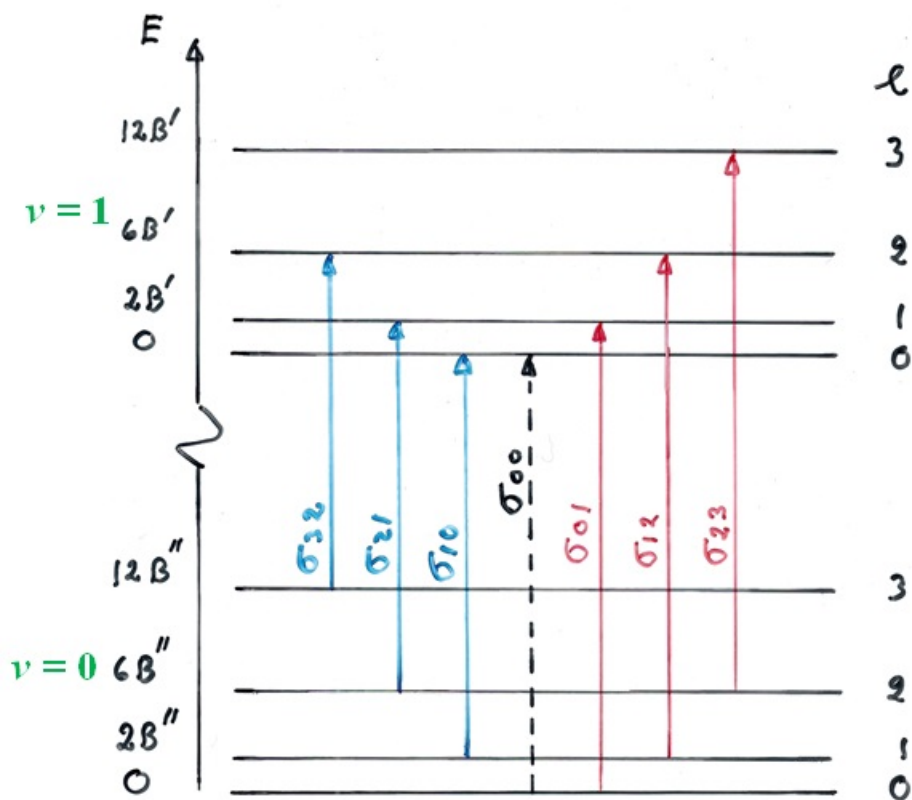
$$E_{rot} = \frac{\hbar^2}{2\mu R_0^2} \cdot \ell \cdot (\ell+1) \equiv B \cdot \ell \cdot (\ell+1)$$

Selection rule for rotation: $\Delta\ell = \pm 1$.

$\Delta\ell = +1$ is called R-branch $\Delta\ell = -1$ is called P-branch

The figure shows part of an absorption spectrum for CO, in the $0 - 2$ vibrational transition, registered using a Fourier-Transform spectrometer (in the figure the absorption spectrum is inverted so that it looks like an emission spectrum)

Using a Gaussian line profile we obtain the wavenumbers:

P_1, i.e. $\ell = 1 \rightarrow \ell = 0$: $4256{,}1248 \text{ cm}^{-1}$

R_1, i.e. $\ell = 0 \rightarrow \ell = 1$: $4263{,}7566 \text{ cm}^{-1}$.

Use this to determine the equilibrium distance between the C and O atoms assuming the moment of inertia is the same in the 2 vibrational levels
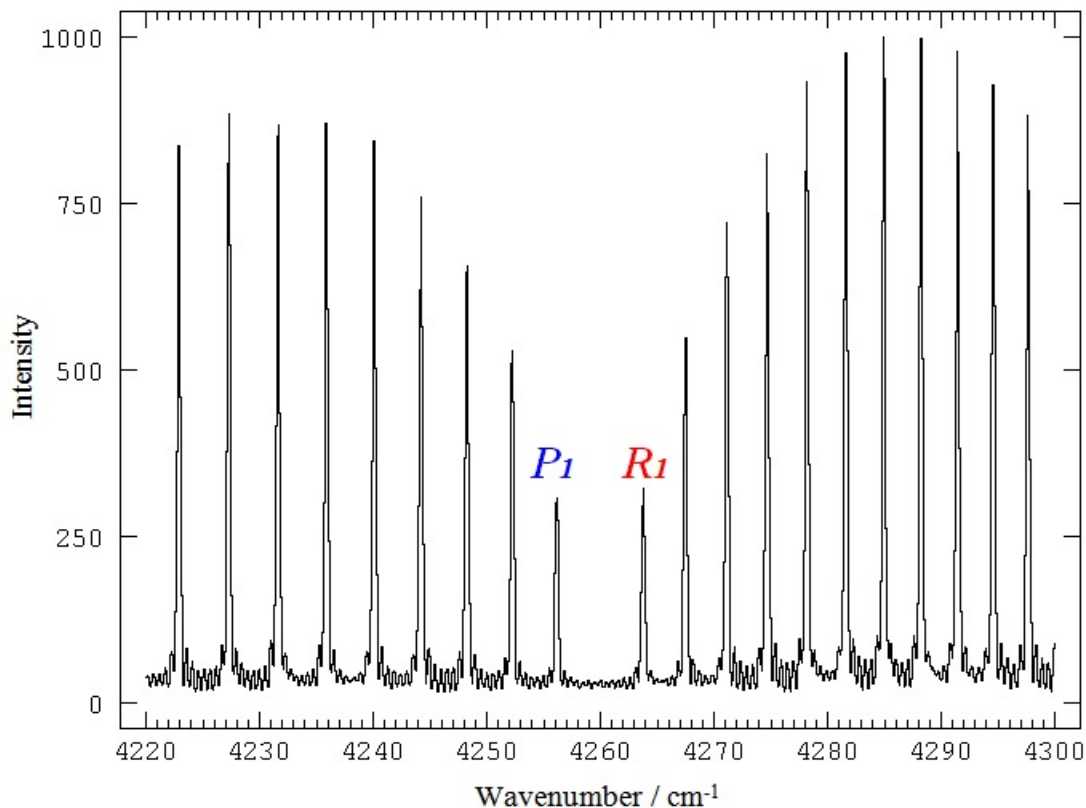
Molecular spectra – selection rules. (SP Ch.6)

For light to be emitted or absorbed the total angular momentum of the molecule has to change since the photon that is emitted/absorbed in a transition carries one unit of angular momentum. Since the vibrational states are not associated with any angular momentum, this implies that the rotational state and/or the electronic state must change. To state selection rules we therefore have to separate three cases.

Pure rotational transitions, i.e. within a vibrational level in a given electronic state. Wavelengths in the microwave range.
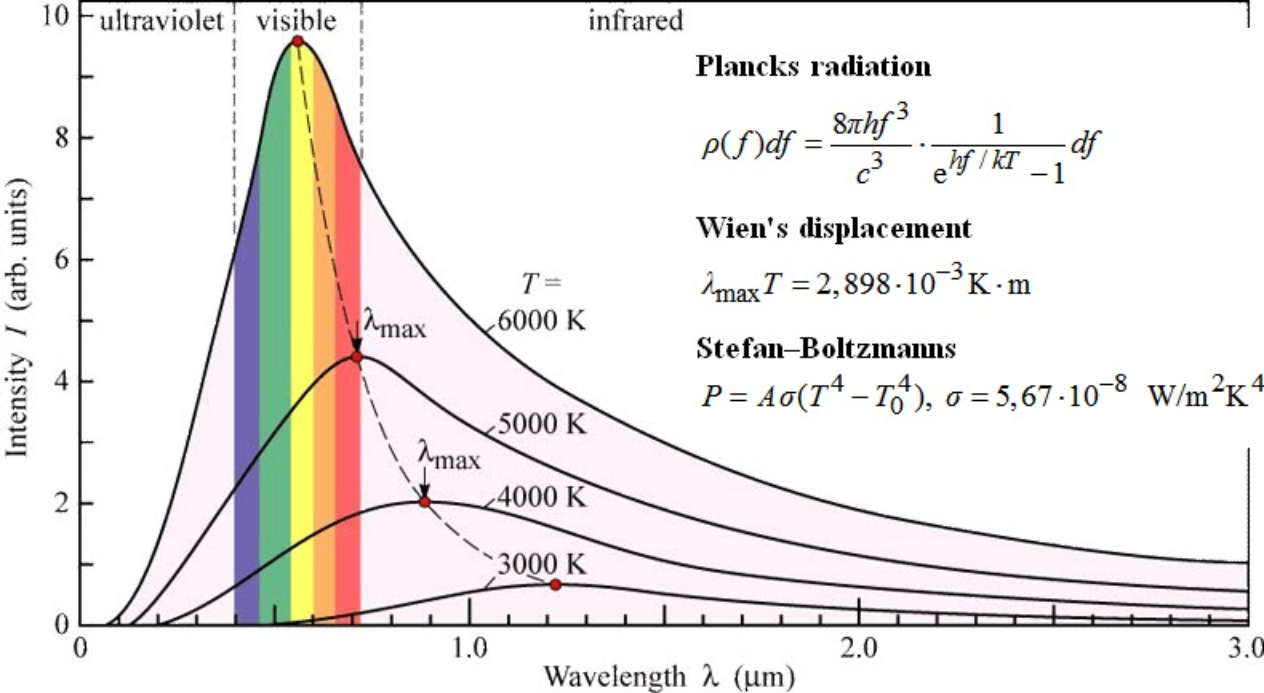
a. Homonuclear molecules (e.g. O_2, N_2), that lack permanent dipole moment, do not have rotational spectra since their rotation cannot interact with electromagnetic radiation.

b. $\Delta\ell = \pm 1$

Rotational-vibrational transitions, i.e. within a given electronic state. Wavelengths in IR.

a. Homonuclear molecules, that lack a permanent dipole moment, do not have rotational-vibrational spectra since their vibration cannot interact with the light.

b. $\Delta v = \pm 1$ (harmonic-oscillator approximation), $\pm 2, \pm 3,...$

c. $\Delta\ell = \pm 1$ (+1 yield R-branch, -1 yield P-branch)

Transitions between electronic states. Wavelengths in visible or UV.

a. Analogous with the situation in atoms, selection rules can be stated for electronic transitions in molecules, utilizing the total eletron angular momentum and spin.

b. $\Delta v = 0, \pm 1, \pm 2, \pm 3,...$

c. $\Delta\ell = 0, \pm 1$ ($\Delta\ell = 0$ is called the Q-branch)

$\Delta\ell = 0$ is forbidden in a pure rotational-vibrational transition, i.e. within an electronic state but permitted in conjunction with electronic transitions since the total angular momentum of the molecule might be different in the lower and upper electronic state respectively.

Plancks radiation

$$\rho(f)df = \frac{8\pi h f^3}{c^3} \cdot \frac{1}{e^{hf/kT} - 1} df$$

Wien's displacement

$$\lambda_{max} T = 2,898 \cdot 10^{-3} \, \text{K} \cdot \text{m}$$

Stefan–Boltzmanns

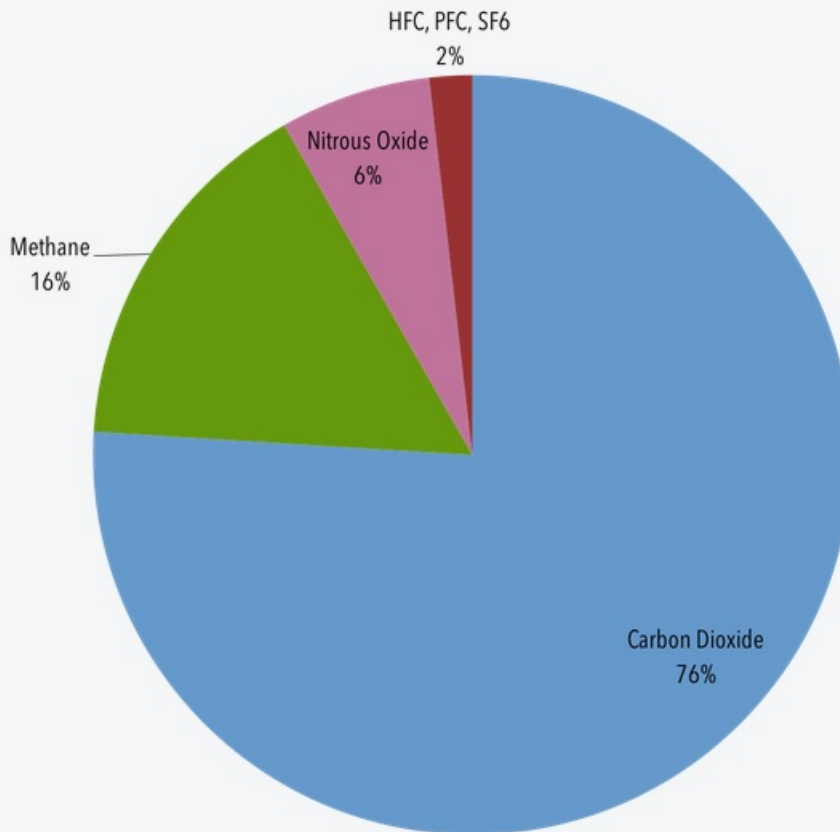$$P = A\sigma(T^4 - T_0^4), \quad \sigma = 5,67 \cdot 10^{-8} \, \text{W/m}^2\text{K}^4$$

The Greenhouse Effect.

The black-body radiation from the Sun (5500 °C) and the Earth (18 °C) and the absorption in the atmosphere in different wavelength regions
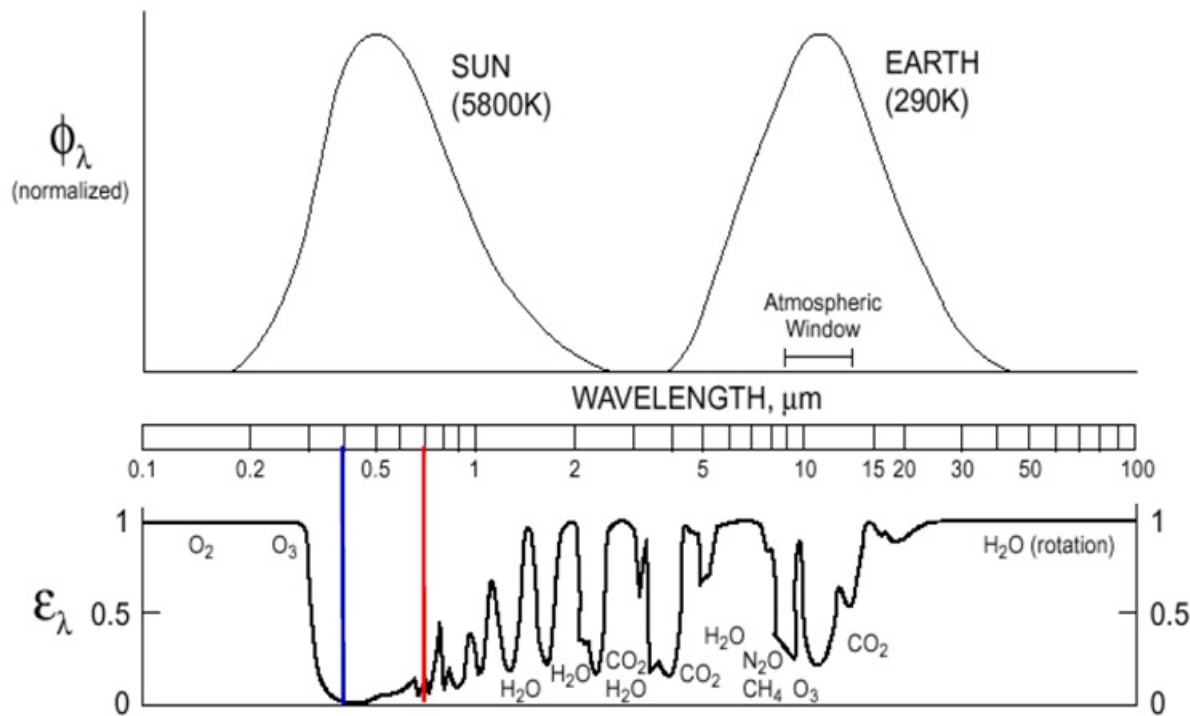
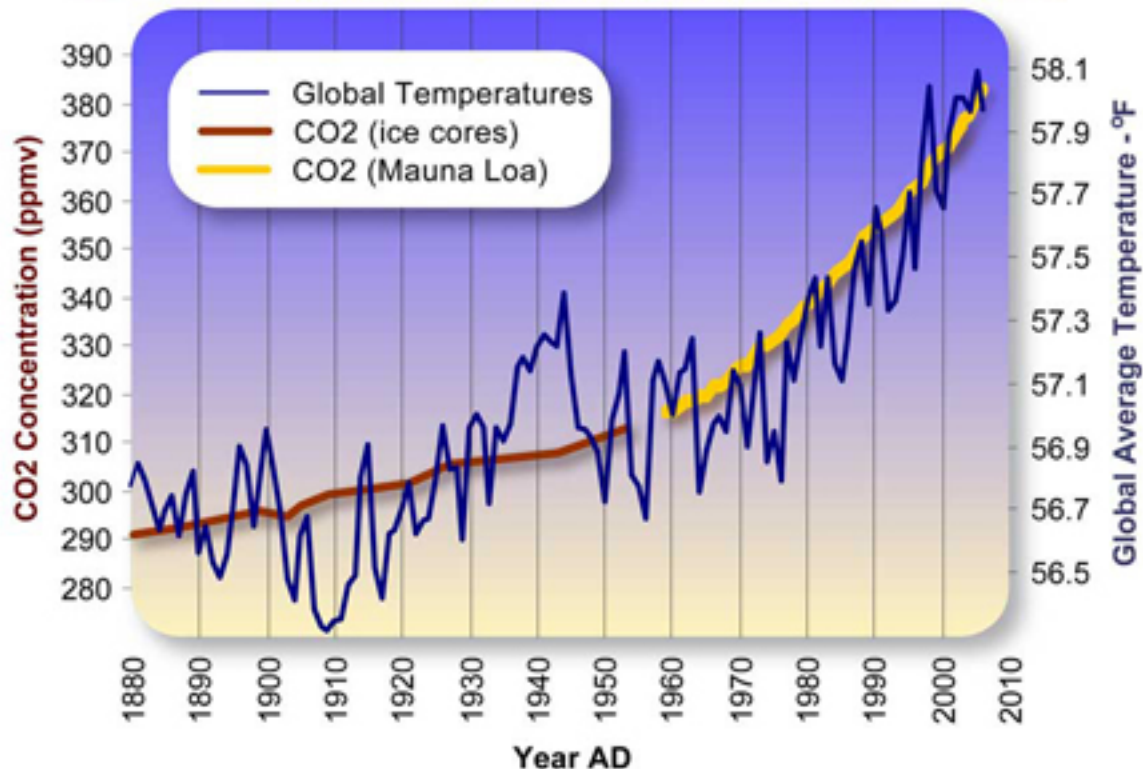# Global Manmade Greenhouse Gas Emissions by Gas, 2015

HFC, PFC, SF6
2%

Nitrous Oxide
6%

Methane
16%

Carbon Dioxide
76%

Global Average Temperature and Carbon Dioxide Concentrations, 1880 - 2006

Legend:
- Global Temperatures
- CO2 (ice cores)
- CO2 (Mauna Loa)

CO2 Concentration (ppmv) — left axis: 280, 290, 300, 310, 320, 330, 340, 350, 360, 370, 380, 390

Global Average Temperature - °F — right axis: 56.5, 56.7, 56.9, 57.1, 57.3, 57.5, 57.7, 57.9, 58.1

Year AD — 1880, 1890, 1900, 1910, 1920, 1930, 1940, 1950, 1960, 1970, 1980, 1990, 2000, 2010

Global Greenhouse Gas Emissions by Economic Sector

Pie chart of Global Greenhouse Gas Emissions by Economic Sector:
- Electricity and Heat Production 25%
- Agriculture, Forestry and Other Land Use 24%
- Industry 21%
- Transportation 14%
- Other Energy 10%
- Buildings 6%