Addition of (all types of) angular momenta.

McMurry Ch 6.3. Exercises: 10 - 11 + 17 (**H3**)

Extremely important in atomic physics!!

Operators: $\hat{J} = \hat{j}_1 + \hat{j}_2.$

Quantum numbers: $J = |j_1 - j_2|, |j_1 - j_2| + 1, \ldots\ldots, j_1 + j_2, \quad M = -J, -J+1, \ldots, J$

Eigenfunctions:

χ_{j_i,m_i} eigenfunction to $\hat{j}_i^2$ and $\hat{j}_{iz}$, $i = 1,2$

$\psi_{j_1,j_2,J,M}$ eigenfunction to $\hat{j}_1^2, \hat{j}_2^2, \hat{J}^2$ and $\hat{J}_z$

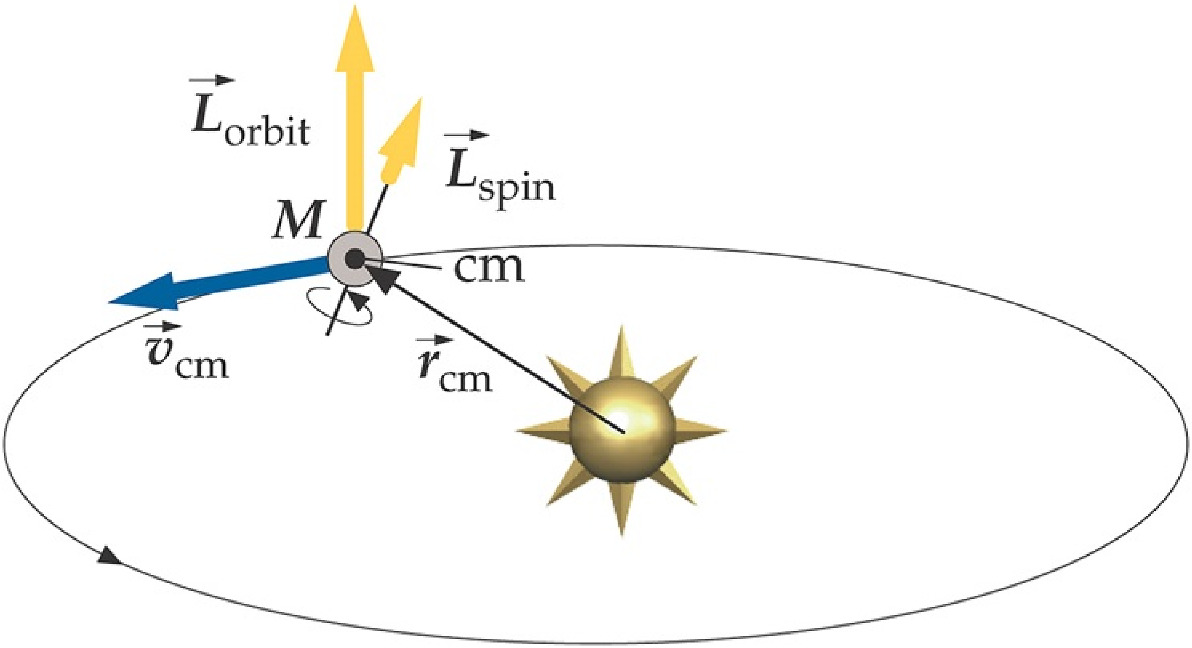
$$\psi_{j_1,j_2,J,M} = \sum_{m_1} C(j_1, m_1, j_2, M - m_1 : J, M) \cdot \chi_{j_1,m_1} \cdot \chi_{j_2,M-m_1}$$

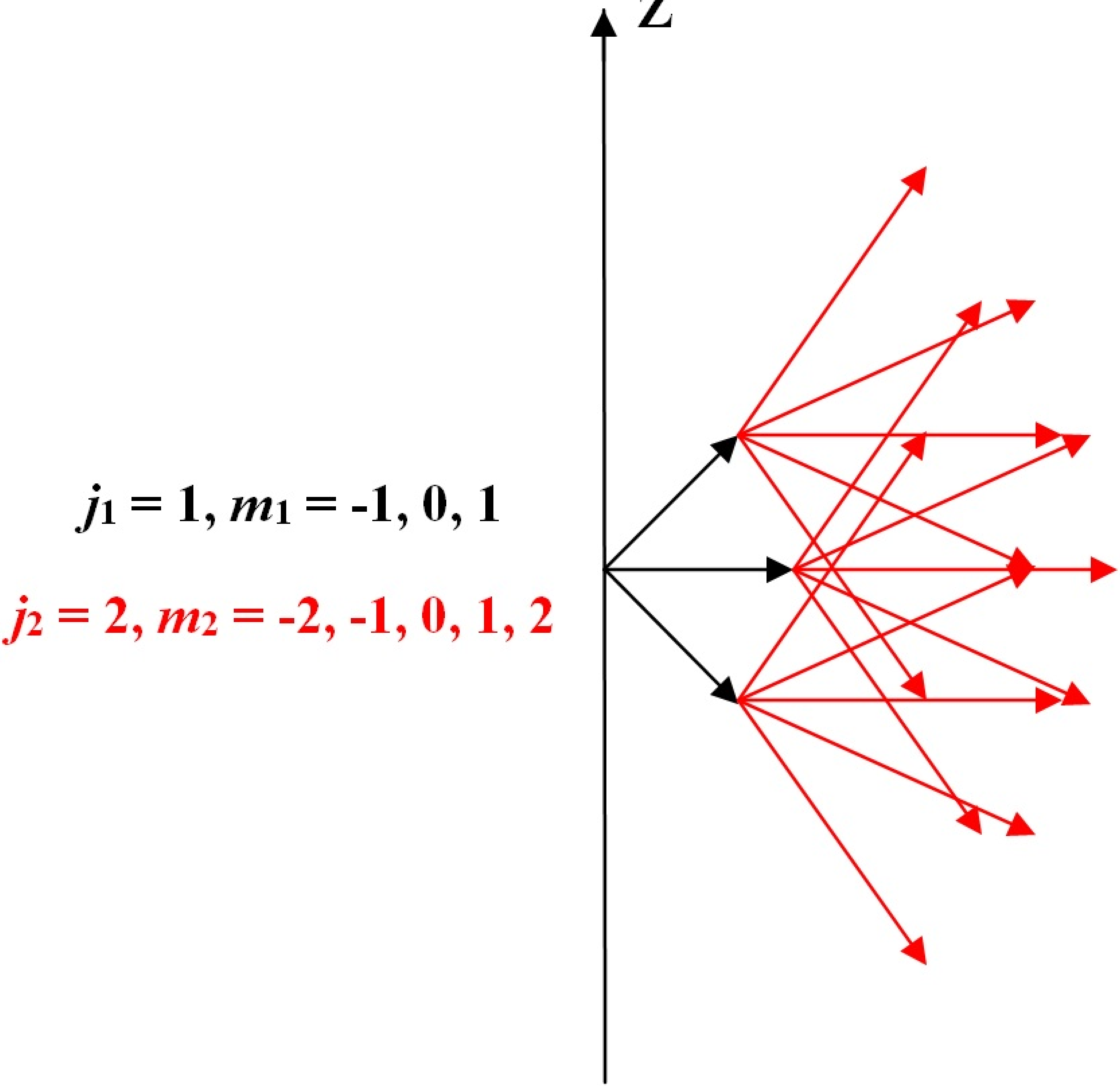
where the C - coefficients are called Clebsch-Gordan, and are given by exact analytical expressions

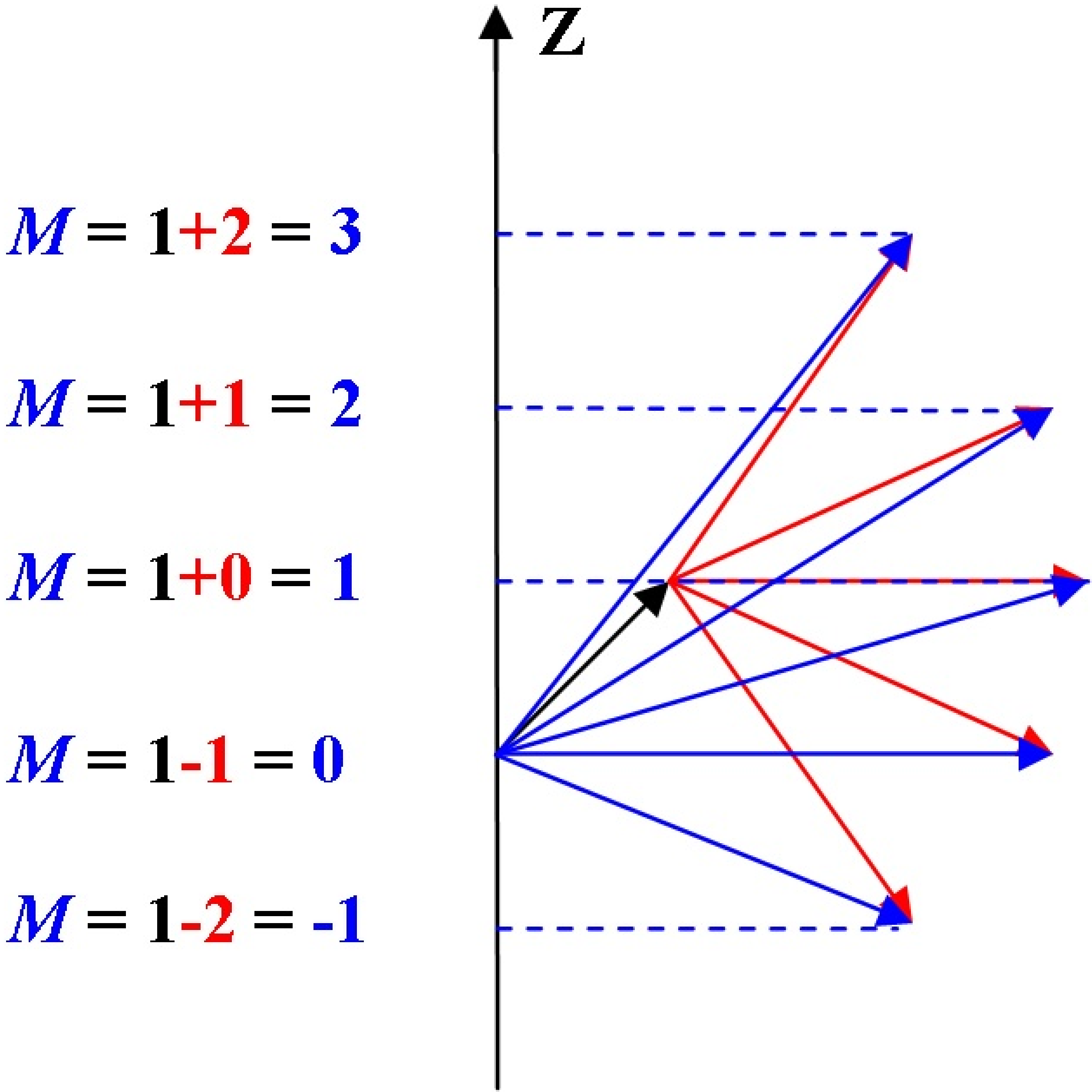
Classical example of the addition of angular momenta.

$$\mathbf{L}_{\text{system}} = \mathbf{L}_{\text{orbit}} + \mathbf{L}_{\text{spin}} = \mathbf{r}_{\text{cm}} \times M \cdot \mathbf{v}_{\text{cm}} + \mathbf{L}_{\text{spin}} =$$

$$\mathbf{r}_{\text{cm}} \times M \cdot \mathbf{v}_{\text{cm}} + I \cdot \boldsymbol{\omega}$$




Z
$j_1 = 1, m_1 = -1, 0, 1$
$j_2 = 2, m_2 = -2, -1, 0, 1, 2$


Z
M = 1+2 = 3
M = 1+1 = 2
M = 1+0 = 1
M = 1-1 = 0
M = 1-2 = -1

Coupling of two angular momenta

Let χ_{j_i,m_i} be eigenfunctions of $\hat{j}_i^2$ and $\hat{j}_{iz}$ for $i=1$ and 2, and $\hat{J}=\hat{j}_1+\hat{j}_2$.

$$J = |j_1 - j_2|, |j_1 - j_2| + 1, \ldots\ldots, j_1 + j_2$$

The eigenfunctions of $\hat{J}^2$ and $\hat{J}_z$ are then obtained through:

$$\psi_{j_1,j_2,J,M} = \sum_{m_1} C(j_1, m_1, j_2, M - m_1 : J, M) \cdot \chi_{j_1,m_1} \cdot \chi_{j_2,M-m_1}.$$

where the C-factors are called Clebsch-Gordan coefficients.

$\psi_{j_1,j_2,J,M}$ is an eigenfunction of $\hat{j}_1^2$, $\hat{j}_2^2$, $\hat{J}$, and $\hat{J}_z$

Clebsch-Gordan coefficients, both in exact analytical form and also numerically, may be found in tables or obtained from the net, e.g at:
http://personal.ph.surrey.ac.uk/~phs3ps/cgjava.html