Roadmap transitions

- **Einstein coefficients A_{21}, B_{12} and B_{21} (H2, #31)**
  Foot Ch 1.7, SP 7.4, 7.5
- **Basic conditions for laser action**
- **Much more about lasers (not covered here):**

  Spectrophysics Ch. 14

  Wikipedia,

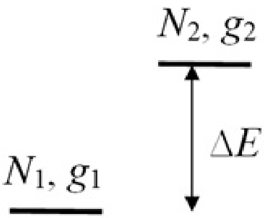
  Atomic Physics Home page (Popular description in Swedish)

  http://www.atomic.physics.lu.se/research/,

  Lecture by Prof Anne L'Huillier (Monday 11/2)

  Many elective courses given by the division
- **Lifetime and Intensity**
- **Selection rules**
  Foot Table 5.1, SP 2.4.4
- **Relative intensities in *LS* multiplets**
  (H2, #33) and two-electron lab

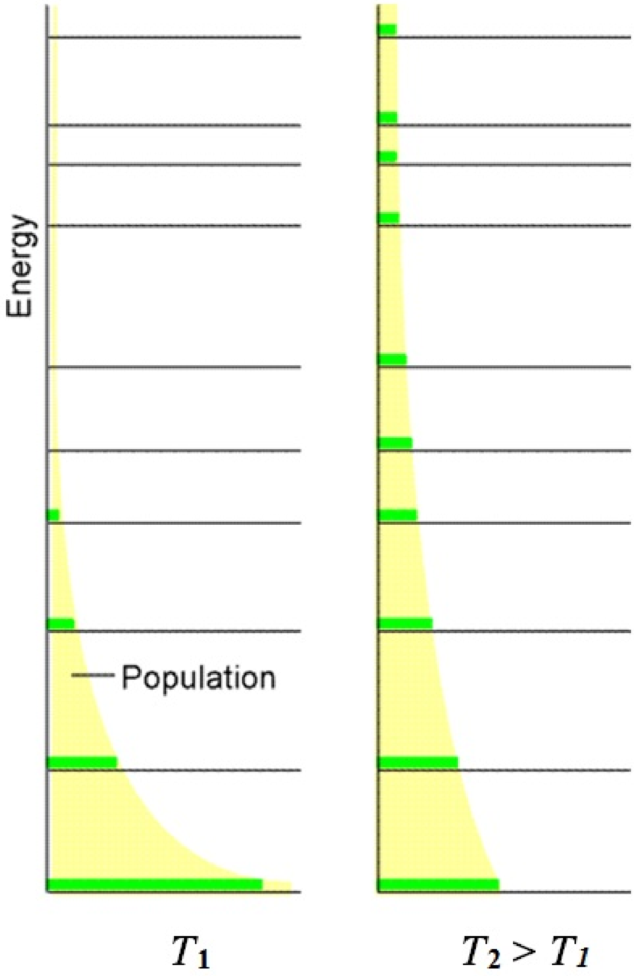
Boltzmann population distribution at thermal equilibrium



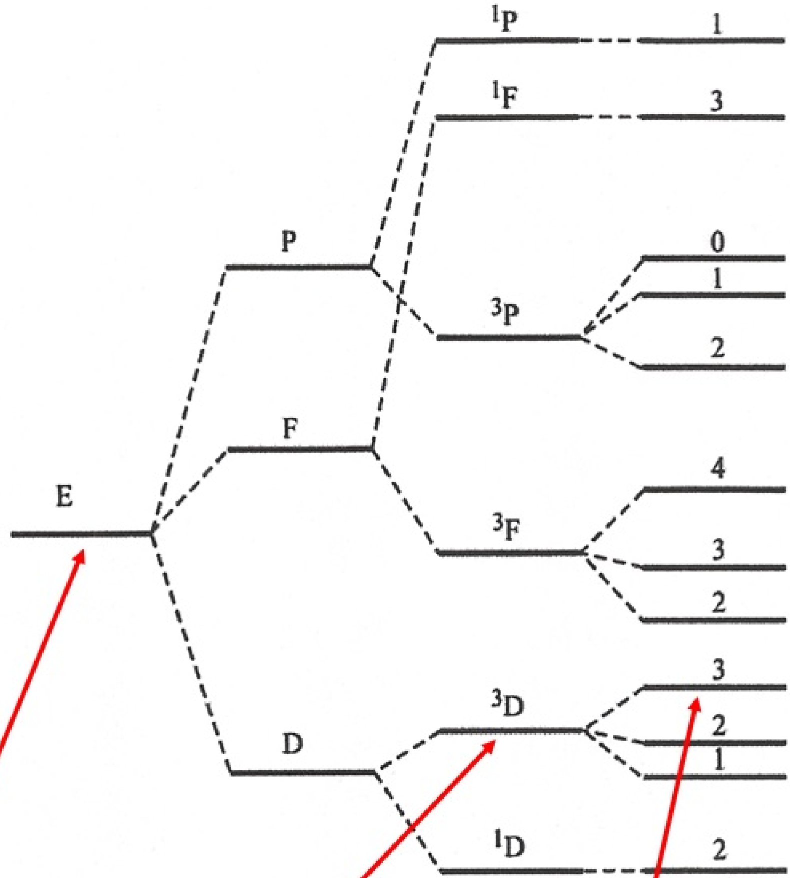
$$\frac{N_2}{N_1} = \frac{g_2}{g_1} \cdot e^{-\Delta E/kT}$$

$g =$ statistical weight

e.g. $2J+1$ or $2F+1$



Degeneracy – statistical weight, *g*, in a pd-configuration.



$$g(E) = 60,$$

$$g(^3D) = (2L+1)\cdot(2S+1) = 15$$

$$g(^3D_3) = 2J+1 = 7$$

Planck's Radiation Law

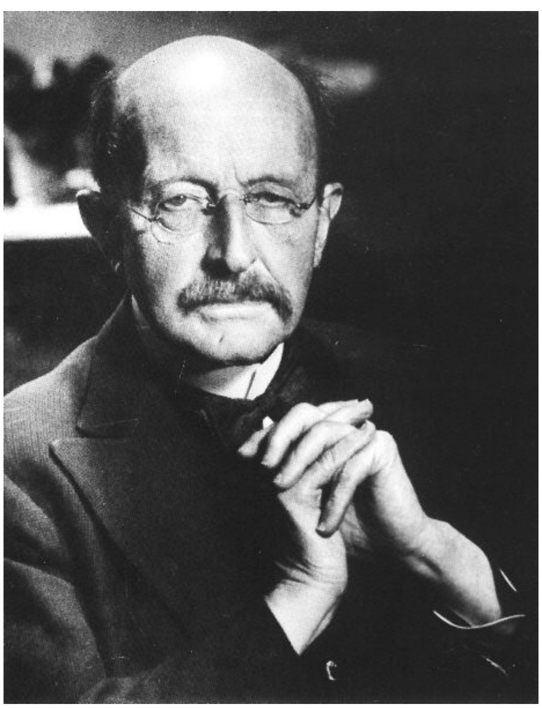
Assume that light can only be absorbed or emitted in discrete quantities (photons) where the energy depends on frequency as:

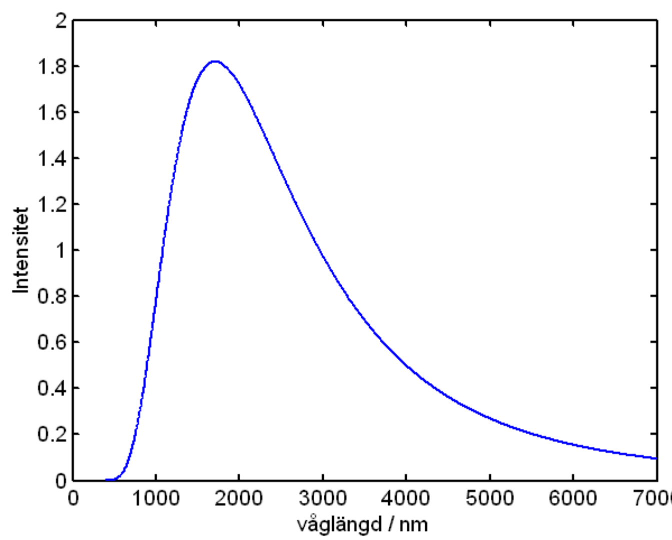
$$E = h \cdot f = h \cdot \frac{c}{\lambda},\ h = 6{,}62 \cdot 10^{-34}\ \text{Js}$$

The energy density per frequency interval, ρ, is then given by:

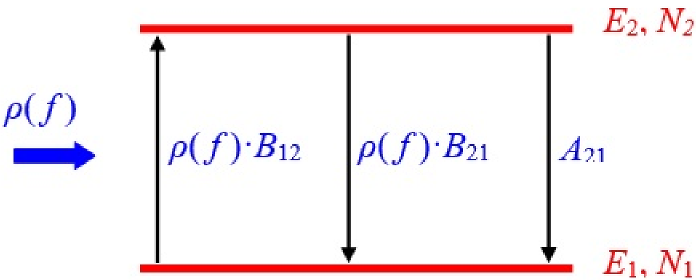
$$\rho(f) = \frac{8\pi h f^3}{c^3} \cdot \frac{1}{e^{hf/kT} - 1},$$

$$[\rho] = 1\ \text{J/(m}^3\times\text{Hz)} = 1\text{Js/m}^3$$





Einstein coefficients



$$A_{21}N_2 + \rho B_{21}N_2 = \rho B_{12}N_1 \Rightarrow ...... \Rightarrow \begin{cases} g_2 B_{21} = g_1 B_{12} \\ g_2 A_{21} = \dfrac{8\pi h f^3}{c^3} g_1 B_{12} \end{cases}$$

(Exercise 32)

$$\text{If } g_1 = g_2 \text{ then } \begin{cases} B_{21} = B_{12} \\ A_{21} \sim f^3 \cdot B_{21}, \sim f^3 \cdot B_{12} \end{cases}$$

Consequences:

- Drive hard and *saturate* a transitions, i.e. $N_1 = N_2$
- Laser actions requires $N_2 > N_1$ i.e. an *inverted* population
- Difficult to obtain laser action at short wavelengths due to the f^3 scaling

Selection rules and metastable levels in He

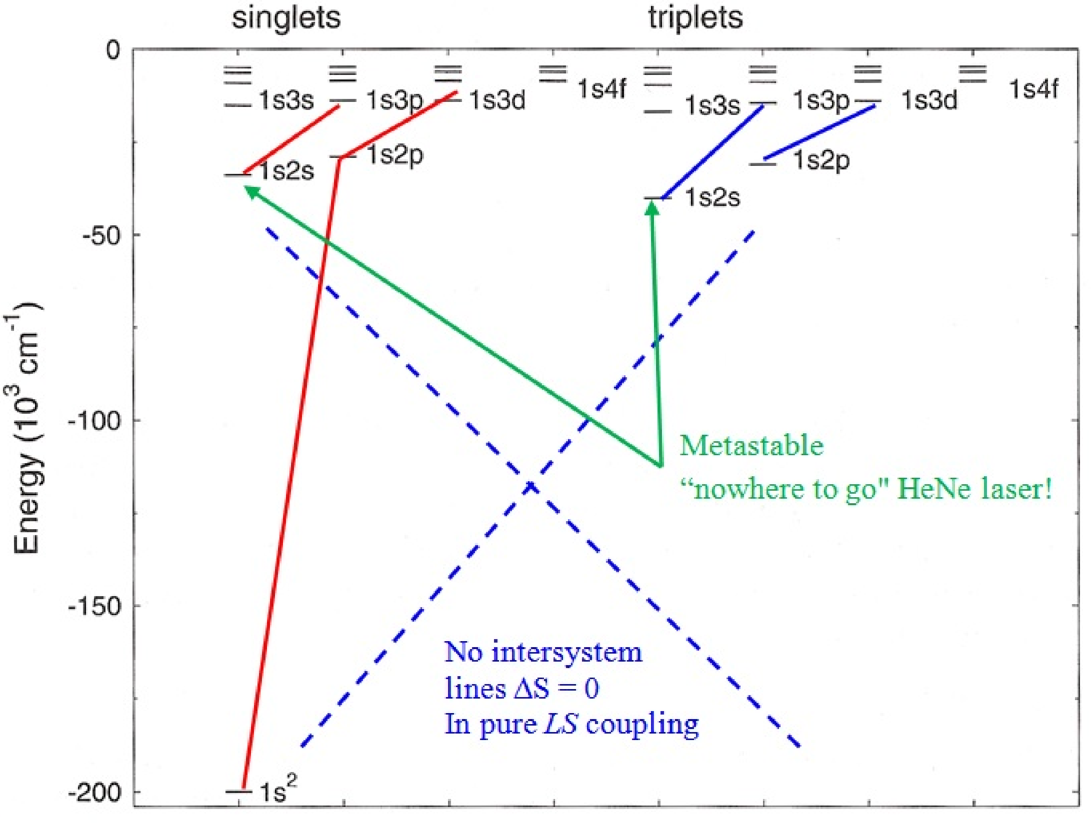


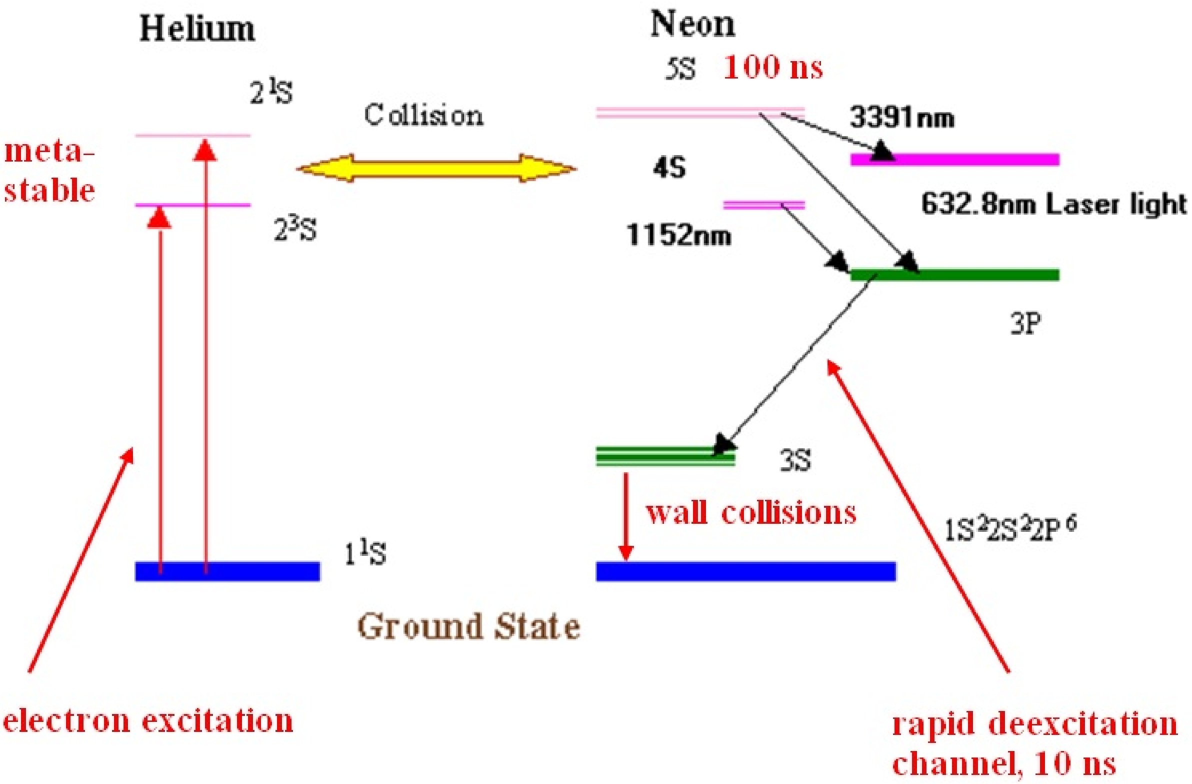
Fig. 2.9. The energy level structure of He.

Observations of so-called spin forbidden transitions, i.e. where $\Delta S \neq 0$, is one sign of intermediate coupling effects.
The $1s^2\ ^1S_0$ - $1s2p\ ^3P_1$ has indeed been observed in all He-like systems.

Laser problem 1: Inverted population

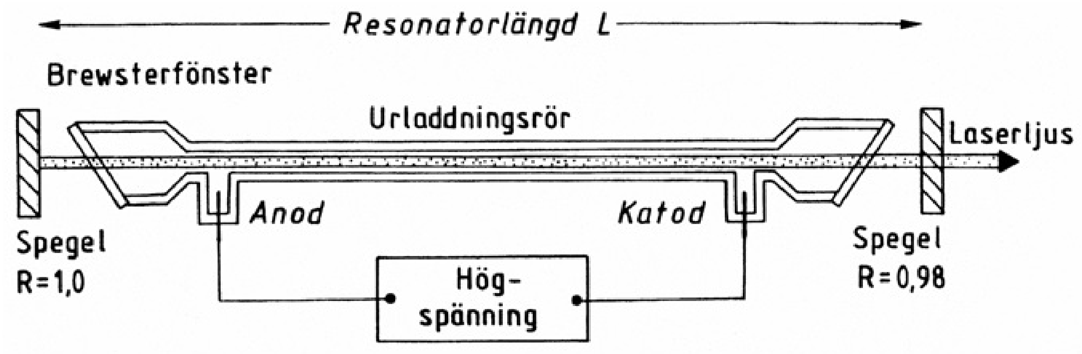
Always some ”trick”.

For example optical pumping or the HeNe-scheme. The latter uses a near coincidence in energy between 2s ^{1,3}S in He and 4s and 5s in Ne which opens a selective collisional excitation of the latter levels in Ne, thereby obtaining an inverted population relative to lower lying levels.



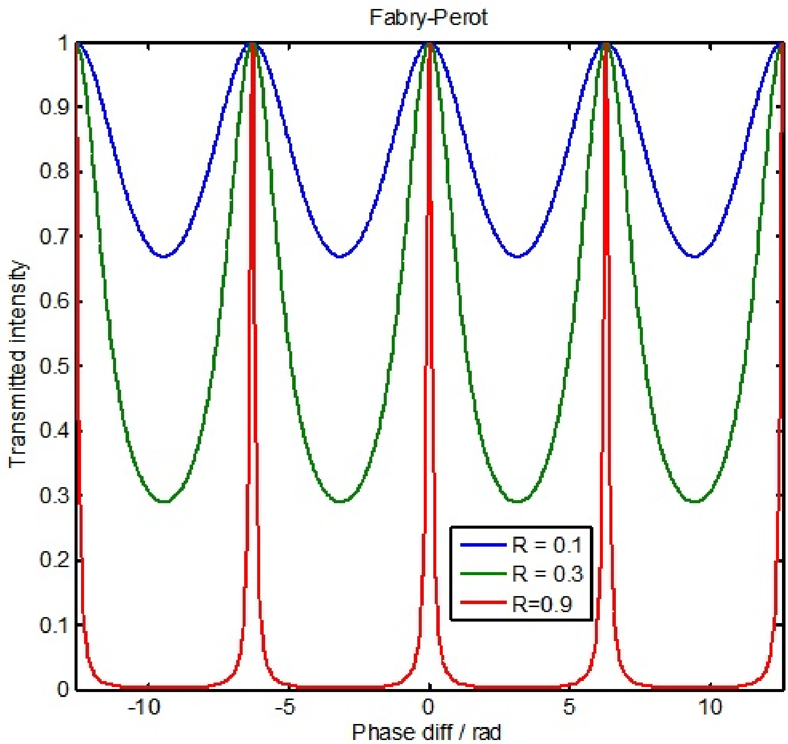
Laser problem 2: High intensity

(Fabry-Perot lecture in lab preparation)



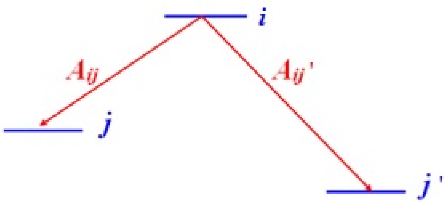
$$\Delta\sigma_{fsr} = \frac{1}{2 \cdot L \cdot n},$$

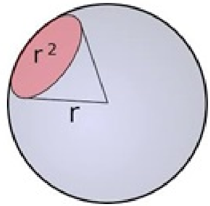
$$(\Delta\delta)_{\min} = 2 \cdot \frac{1-R}{\sqrt{R}}$$



Spectral line intensity.

$$n_{ij} = A_{ij} \cdot N_i$$





Where n_{ij} is the number of emitted photons per second in 4π steradian (full sphere).

The number of **detected** photons, the *intensity,* is then

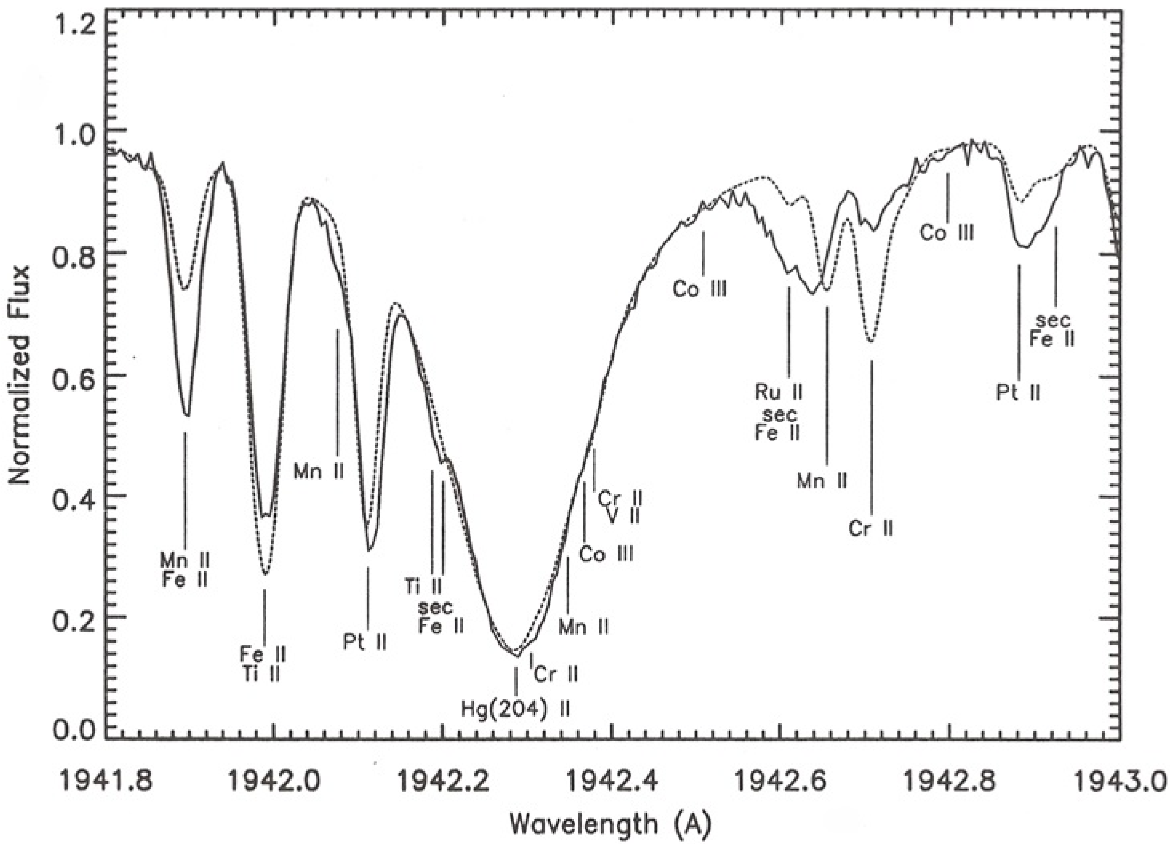
$$\boxed{I_{ij} = \varepsilon(\lambda_{ij}) \cdot A_{ij} \cdot N_i}$$

where $\varepsilon(\lambda)$ takes care of the solid angle subtended by the detector and all, wavelength dependent, detector efficiencies. Here the intensity is measured in number of photons per second and unit area, meaning that the normal unit W/m^2 is obtaind by multiplying by the energy *hf* of one photon.


Eta Carinae
Almost all information we have about our surroundings comes from the analysis of light

Astrophysical application

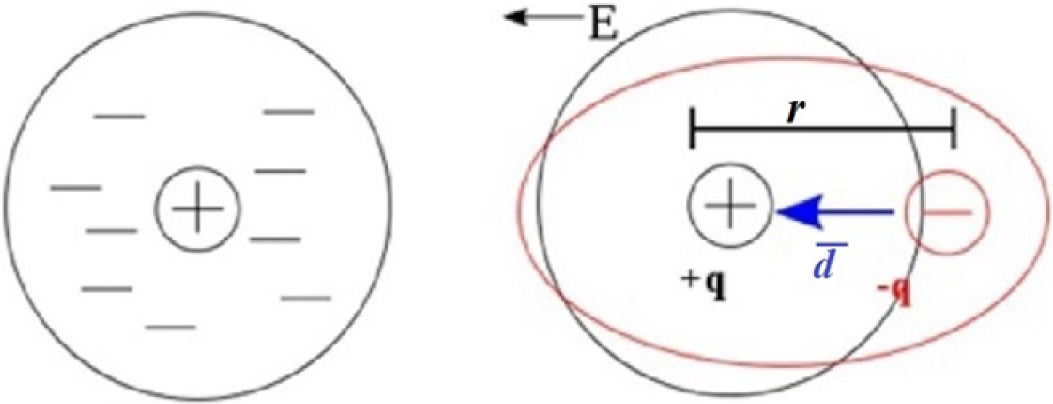
Chi Lupi is a blue giant star in the constellation Lupus, 195 light years away



Isotope anomaly of Hg in the atmosphere of χ – Lypi.
Leckrone, Wahlgren and Johansson Ap J 377, L37 (1991)

Absorption - classical.

The incoming light (electric field) induces an oscillating electric dipole moment $\bar{d} = -q \cdot \bar{E}$

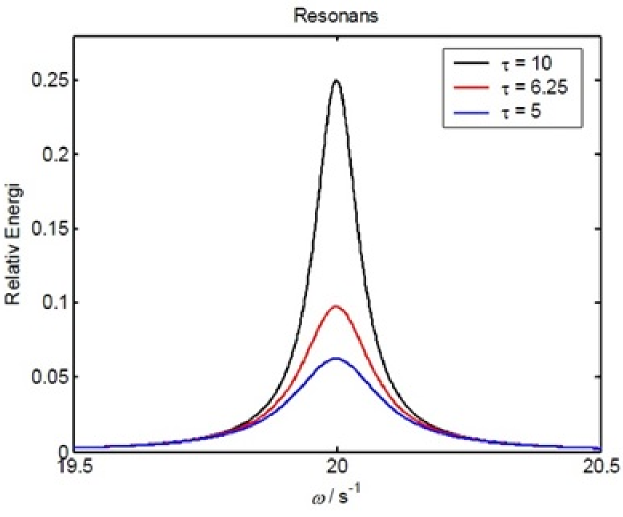


Damped and driven harmonic oscillator

$$\frac{d^2 r}{dt^2} + \frac{b}{m} \cdot \frac{dr}{dt} + \omega_0^2 \cdot r = \frac{qE_0}{m} \cdot \cos \omega t$$

$$r(t) = A \cdot \cos(\omega t - \delta)$$

$$A = \frac{qE_0}{\sqrt{m^2(\omega_0^2 - \omega^2)^2 + b^2\omega^2}}$$



Selection rules E1 (electric dipole) transitions

$\Delta J = 0, \pm 1$ not 0 to 0

Exercise 17

Only one electron can change orbital, i.e. $n\ell$

Highly unlikely that two electrons would rearrange themselves simultaneously

$\hat{r} =$ one-electron operator

$\Delta \ell = \pm 1$

$\hat{r}$ has odd parity and $Y_{\ell,m}(\theta,\varphi)$ has $(-1)^{\ell}$

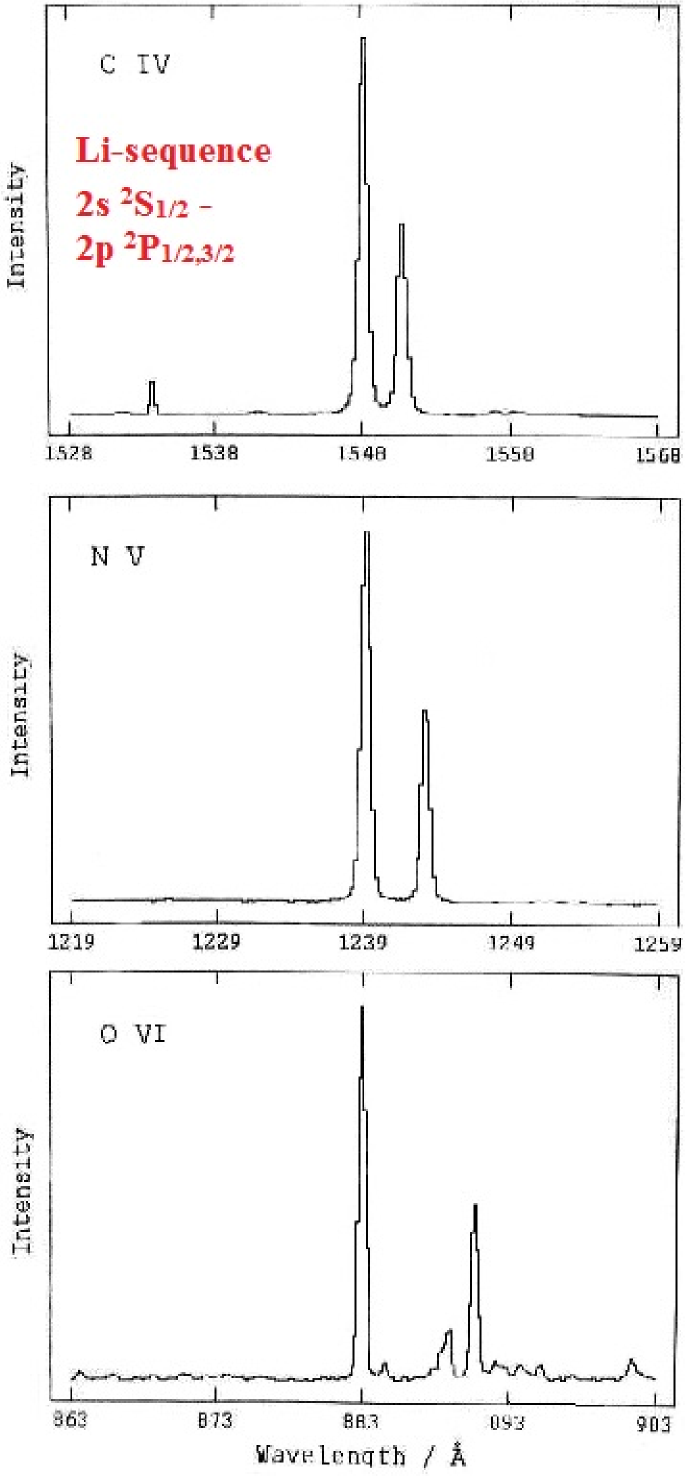
If perfect LS-coupling, i.e real states = basis states

$\Delta S = 0$

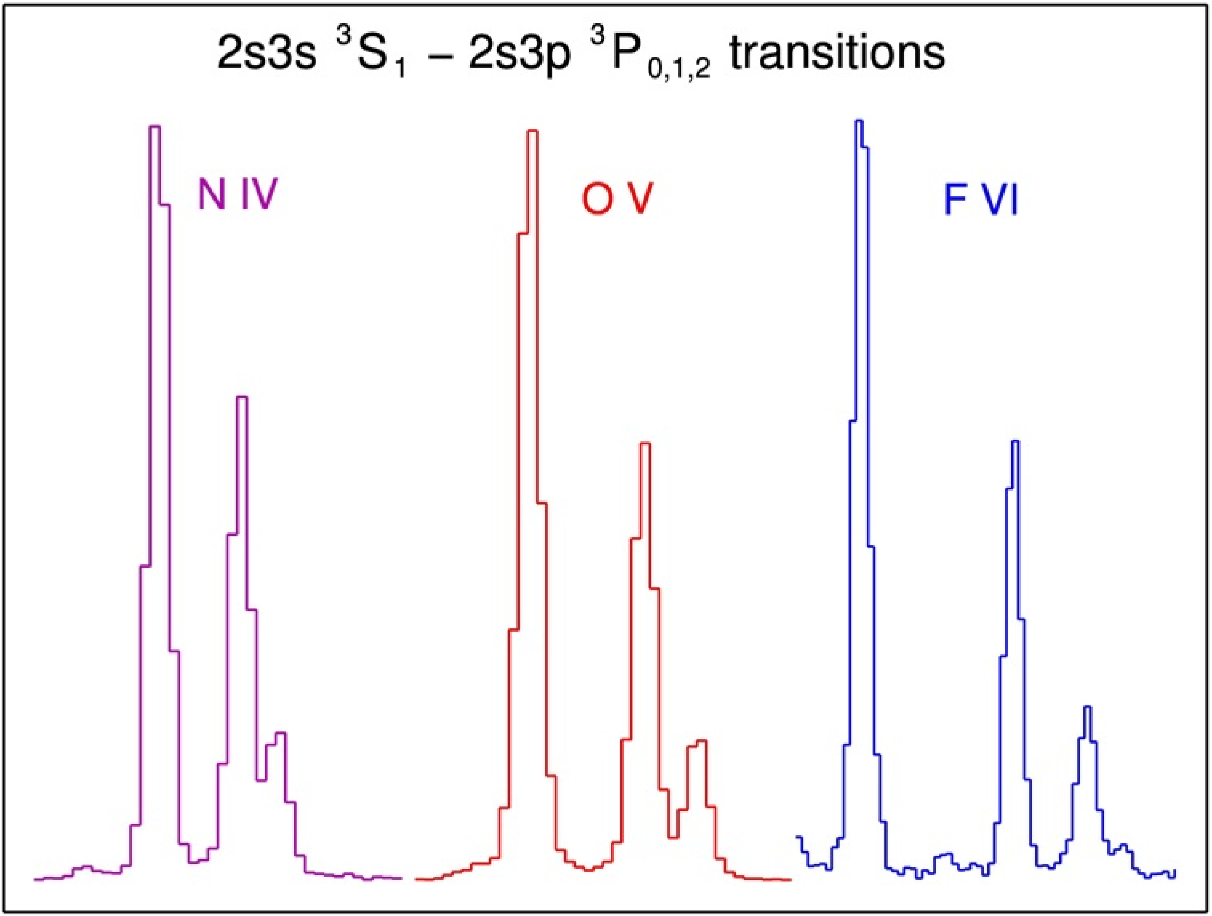
$\hat{r}$ does not include spin, thus can't change it

$\Delta L = 0, \pm 1$ ej 0 till 0

Follows from ΔJ and ΔS


C IV
Li-sequence
2s $^2S_{1/2}$ -
2p $^2P_{1/2,3/2}$
Intensity
1528
1538
1548
1558
1568
N V
Intensity
1219
1229
1239
1249
1259
O VI
Intensity
863
873
883
893
903
Wavelength / Å

Relative intensities in the Be-sequence



Relative intensities in some LS multiplets

The (normally) very intense "diagonal" in the multiplets is shown in red

	$^2S_{1/2}$	$^2P_{1/2}$	$^2P_{3/2}$	$^2D_{3/2}$	$^2D_{5/2}$
$^2P_{1/2}$	1	2	1	5	---
$^2P_{3/2}$	2	1	5	1	9

	$^2D_{3/2}$	$^2D_{5/2}$	$^2F_{5/2}$	$^2F_{7/2}$	$^2G_{7/2}$	$^2G_{9/2}$
$^2F_{5/2}$	14	1	20	1	27	--
$^2F_{7/2}$	--	20	1	27	1	35

	$^3S_1$	$^3P_0$	$^3P_1$	$^3P_2$	$^3D_1$	$^3D_2$	$^3D_3$
$^3P_0$	1	--	4	--	20	--	--
$^3P_1$	3	4	3	5	15	45	--
$^3P_2$	5	--	5	15	1	15	84

	$^3D_1$	$^3D_2$	$^3D_3$
$^3D_1$	81	27	--
$^3D_2$	27	125	28
$^3D_3$	--	28	224

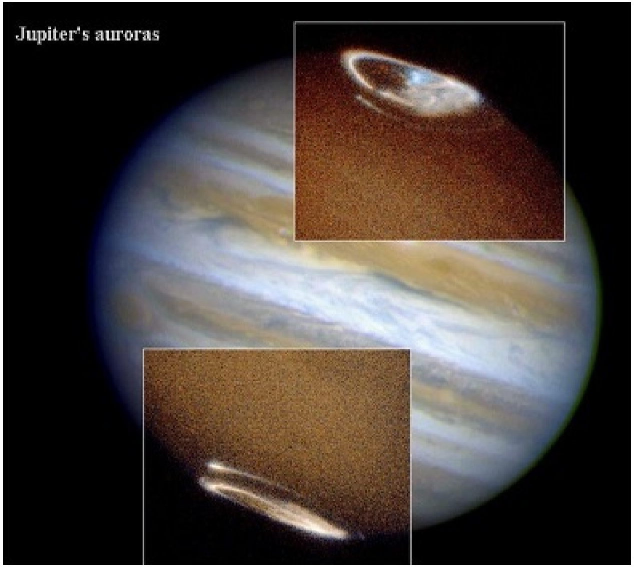
	$^3D_1$	$^3D_2$	$^3D_3$	$^3F_2$	$^3F_3$	$^3F_4$	$^3G_3$	$^3G_4$	$^3G_5$
$^3F_2$	189	35	1	640	80	--	720	--	--
$^3F_3$	--	280	35	80	847	81	63	945	--
$^3F_4$	--	--	405	--	81	1215	1	63	1232

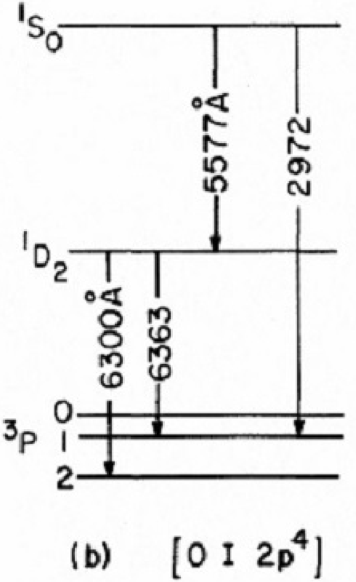
Relative intensities in a ^3D - ^3F multiplet and the LS sum rules

	$^3D_1$	$^3D_2$	$^3D_3$	Σ int.	$g_L = 2J+1$	(Σ int.)/g_L
$^3F_2$	189	35	1	225	5	45
$^3F_3$		280	35	315	7	45
$^3F_4$			405	405	9	45
Σ int.	189	315	441			
$g_U = 2J+1$	3	5	7			
(Σ int.)/ g_U	63	63	63			

The sum of all intensity TO a given LOWER level is proportional the statistical weight ($2J+1$) of the level.
The constant is the same for all levels of the lower LS term.

The sum of all intensity FROM a given UPPER level is proportional the statistical weight of the level.
The constant is the same for all levels of the upper LS term


Jupiter's auroras


$^1S_0$
$^1D_2$
5577Å
2972
6300Å
6363
^3P
0
1
2
(b) [O I $2p^4$]