Solutions Atomic Physics 2

1. Probability distributions without any zero points (r > 0), i.e. wavefunctions with n - ℓ - 1 = 0 zero crossings, correspond to orbitals with the maximum ℓ for given n, i.e. 1s, 2p, 3d, 4f, 5g,.... For these orbitals it so happens that the radial position of the maximum corresponds to the radius of a circular Bohr orbit with the same main quantum number n:

$$r = a_0 \cdot \frac{n^2}{Z}.$$

From the figure we find the maximum at $r = 16a_0$ and thus $n = 4$.
Hence the orbital is 4f.

2. The center-of-gravity energy for $4p\ ^2$P is

$$E = \frac{1}{6}(2 \cdot 12985.2 + 4 \cdot 13042.9) = 13023.7\ \text{cm}^{-1}$$

The term value is then $T = E_{\text{limit}} - E$ =21986.1 cm^{-1} and the quantum defect

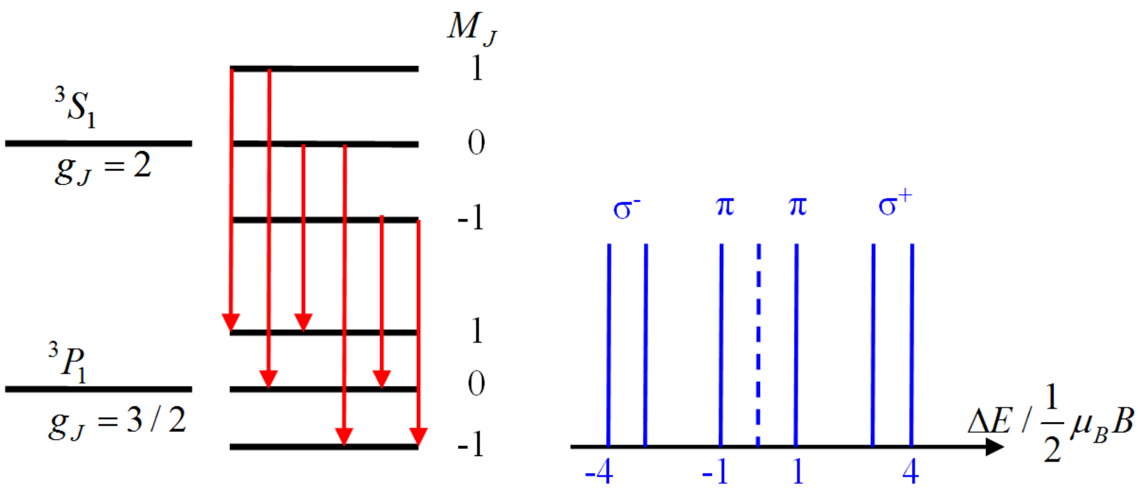
$$\delta = 4 - \sqrt{\frac{R_\infty \cdot 1^2}{T}} = 1.766.$$

Assuming δ to be constant in the p-series we get

$$T(20p\,^2\text{P}) = \frac{R_\infty \cdot 1^2}{(20-\delta)^2} = 330.0\ \text{cm}^{-1} \Rightarrow E(20p\,^2\text{P}) = 34679.7\ \text{cm}^{-1} \Rightarrow \lambda(4s - 20p) = 2883.5\ \text{Å}.$$

The experimental energy of 20p is 34681.7 cm^{-1}.

3ab. The Zeeman components in the $^3P_1$ - $^3S_1$ transition are shown below with their polarization state, when viewed viewed perpendicularly, indicated. Note that 0 → 0 is not possible since ΔJ = 0.



3c. The minimum energy separation is

$$\Delta E_{\min} = \frac{1}{2}\mu_B B \Rightarrow \Delta f_{\min} = \frac{\mu_B B}{2h} = 3.5 \text{ GHz} \Rightarrow \Delta\lambda_{\min} = \frac{\lambda^2}{c}\Delta f_{\min} = 0{,}027 \text{ Å}.$$

4a. If no other effects limit the experimental resolution the limit is set by the natural line width. $\Delta f_N = \frac{1}{2\pi}(\frac{1}{\tau_1}+\frac{1}{\tau_2}) = \frac{1}{\pi\tau}$ if the 2 lifetimes are equal.

With $\Delta f_{\min} = 3.5$ GHz we get $\tau > 91$ ps.

(Using $\Delta\lambda_{\min} = 0{,}05 \text{ Å} \Rightarrow \Delta f_{\min} = 6.5 \text{ GHz} \Rightarrow \tau > 48.9 \text{ ps.}$)

4b. If the limit is set by the Doppler width instead we get:

$$\Delta f_D = C\cdot\frac{c}{\lambda}\cdot\sqrt{\frac{T}{M}} \Rightarrow T < M\cdot(\frac{\Delta f_{\min}\cdot\lambda}{c\cdot C})^2 =$$

$$112 \text{ g/mol}\cdot(\frac{3.5\cdot 10^9 \text{ s}^{-1}\cdot 4800\cdot 10^{-10} \text{ m}}{3\cdot 10^8 \text{ m/s}\cdot 7.16\cdot 10^{-7} \text{ (g/(mol}\cdot\text{K))}^{0.5}})^2 = 6851 \text{ K}$$

$\Delta\lambda_{\min}$=0.05 Å $\Rightarrow T = 23600$ K instead.

5a. From the radiation balance: $(A_{21}+\rho(f_{21})B_{21})\cdot N_2 = \rho(f_{21})B_{12}\cdot N_1$ we find that if the number of stimulated photons should exceed the number of absorbed we must have:

$$\rho(f_{21})B_{21}\cdot N_2 > \rho(f_{21})B_{12}\cdot N_1 \Leftrightarrow \frac{g_1}{g_2}B_{12}\cdot N_2 > B_{12}\cdot N_1 \Leftrightarrow g_1 N_2 > g_2 N_1.$$

If $g_1 = g_2$ this reduces to the more direct condition of an inverted population $N_2 > N_1$.

5b. When the atom absorbs a photon that carries a momentum $p = h/\lambda$ is must change its velocity (Δv) so that the total momentum is conserved.

$\bar{p}_\gamma + \Delta\bar{p}_{\text{atom}} = 0,$ i.e. the velocity change by: $\Delta v\cdot M = \frac{h}{\lambda}$.

6. $\ell_1 = 1, s_1 = 1/2 \Rightarrow j_1 = 1/2, 3/2.$

$j_1 = 1/2, \ell_2 = 2 \Rightarrow K = 3/2,\ 5/2.$ $\quad j_1 = 3/2, \ell_2 = 2 \Rightarrow K = 1/2,\ 3/2,\ 5/2, 7/2.$

If we now add s_2 = 1/2 we will get levels with $J = K \pm 1/2$.

$\frac{1}{2}[3/2]_{1,2},\ \frac{1}{2}[5/2]_{2,3}$

$\frac{3}{2}[1/2]_{0,1},\ \frac{3}{2}[3/2]_{1,2},\ \frac{3}{2}[5/2]_{2,3},\ \frac{3}{2}[7/2]_{3,4}.$

(Note that we get the same *number* of levels and the same set of total *J*: s as in *LS*- or *jj*-coupling)

This coupling scheme could be a good approximation in a case where the largest interaction is the spin-orbit of the most tightly bound (non-*s*) electron (2p), followed by the electrostatic repulsion between the 2 electrons and finally the very small spin-orbit contribution of the outer, high-ℓ electron (5d). Remember that the spin-orbit energy scales as n^{-3} and ℓ^{-3}.

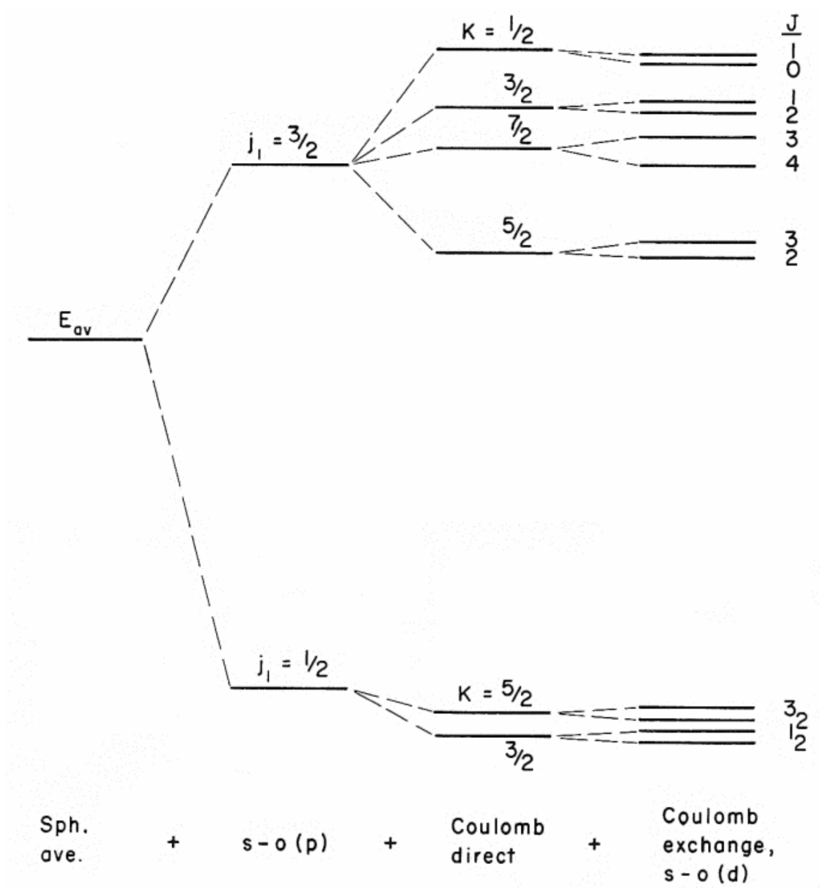


Figure*:* *pd*-configuration in *jK*-coupling