Solutions Atomic Physics 3

1. λ(3s - 3p) = 589.36 nm $\Rightarrow$ $E_{\text{ex}} = 16967.6\ \text{cm}^{-1} \Rightarrow T_{3\text{p}} = E_{\text{ion}} - E_{\text{ex}} = 24481{,}9\ \text{cm}^{-1}$,

   $T_{3\text{s}} = E_{\text{ion}} = 41449.45\ \text{cm}^{-1}$.

   $T = \dfrac{R\xi^2}{(n-\delta)^2}$, $\xi = 1$, $R = 109737\ \text{cm}^{-1} \Rightarrow \delta_{3\text{s}} = 1.3729$ and $\delta_{3\text{p}} = 0.8828$

   Assume the quantum defect to be independent of n for given ℓ then:

   $$\sigma(\text{4s - 4p}) = T_{4\text{s}} - T_{4\text{p}} = R \cdot \left(\frac{1}{(4-\delta_{\text{s}})^2} - \frac{1}{(4-\delta_{\text{p}})^2}\right) = 0{,}041977 \cdot R = 4606.4\ \text{cm}^{-1} \Rightarrow$$

   λ(4s - 4p) =2,17 μm. The experimental value is 2.026 μm.

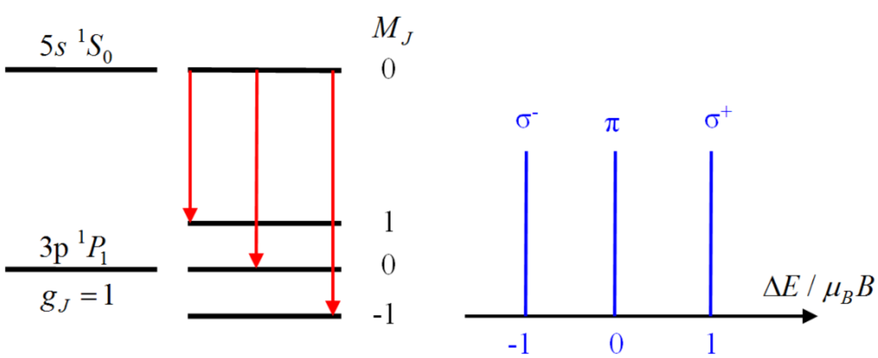
2a. $4s^2$: $^1S_0$. $4s4p$: $^1P_1$, $^3P_{0,1,2}$. $4p^2$: $^1S_0$, $^1D_2$, $^3P_{0,1,2}$. In the last configuration you must consider the equivalent electrons.

2b. $4s^2\ ^1S_0$ - $4s4p\ ^1P_1$.

   $4s4p$ - $4p^2$: $^1P_1$ - $^1S_0$, $^1D_2$. $^3P_0$ - $^3P_1$, $^3P_1$ - $^3P_{0,1,2}$ $^3P_2$ - $^3P_{1,2}$

2c. At the low temperature in the interstellar medium all systems are in their ground state. Thus in absorption only the $4s^2\ ^1S_0$ - $4s4p\ ^1P_1$ line can be expected

3a. The energy levels, Zeeman components and their state of polarization are shown in the figure below.

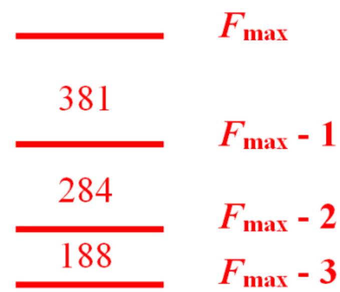


3b. The smallest energy separation is

$\Delta E = B\mu_{\mathrm{B}}$ which corresponds to a frequency difference of $\Delta f = B\mu_{\mathrm{B}} / h$.

$$\Delta\lambda = \frac{\lambda^2}{c}\Delta f = \frac{\lambda^2}{c}\cdot\frac{B\mu_{\mathrm{B}}}{h} \Rightarrow B_{\min} = \frac{\Delta\lambda\cdot c\cdot h}{\lambda^2\cdot\mu_{\mathrm{B}}} =$$

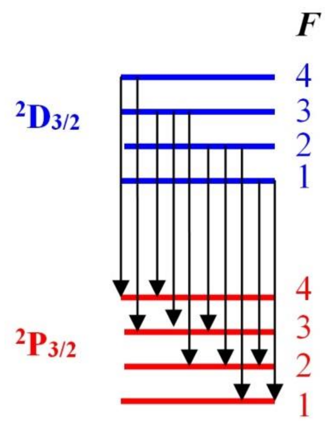
$$\frac{0{,}11\cdot 10^{-10}\,\mathrm{m}\cdot 3{,}0\cdot 10^{8}\,\mathrm{m/s}\cdot 6{,}626\cdot 10^{-34}\,\mathrm{Js}}{(5711.09\cdot 10^{-10}\,\mathrm{m})^2\cdot 9{,}27\cdot 10^{-24}\,\mathrm{J/T}} = 0{,}723\ \mathrm{T}$$



4a. Using the Landé interval rule we get: $(F_{max} / F_{max} - 1)$ = 381/284 = 1,34 => F_{max} = 3,93 ≈ 4.
Test: (F_{max} - 1 / F_{max} – 2) = 284/188 = 1,51 = 3/2.

Thus F = 1,2,3 and 4. With F_{max} = 4 and J = 3/2 we get I = 5/2.

A = 1/3(381/4 + 284/3 + 188/2) = (95,3 + 94,7 + 94,0)/3 = 94,6 MHz.



4b. The 10 allowed transitions are marked in the figure.

5a $L^2:\ 6\hbar^2,\quad L_z:\ -2\hbar,\ -\hbar,\ 0,\ \hbar,\ 2\hbar.$

5b. $Y_{0,0}(\theta,\varphi) = \dfrac{1}{\sqrt{4\pi}}$

5c. $L_+ Y_{1,1}(\theta,\varphi) = 0$. L_+ increases the m quantum number, but sice we are at the maximum value it must be zero

5d. In an isolated system angular momentum is conserved, thus we may use its value as a label or name of a particular state. If we combine the angular momentum of the atomic states before and after the emission or absorption of a photon, with spin angular momentum of 1, we get selection rules for allowed transitions. If we express the kinetic energy part of the Hamilton operator (∇^2) in spherical coordinates the rotational part depends on L^2. And so on........

6. $E = E_{elektron} + E_{vib} + E_{rot} = E_{elektron} + \hbar\omega_0(\nu + \frac{1}{2}) + \frac{\hbar^2}{2\mu r^2} \cdot \ell(\ell + 1)$

Orders of magnitude expressed as energy in eV and in cm^{-1}, wavelength and temperature:

	Electron	Vibration	Rotation
E / cm^{-1}	40000	1500	10
E / eV	5	0.2	0.001
λ / μm	0.3	6	1000
T / K	40000	1500	10